Approximate Solutions for Fractional Gas Dynamics and Convection-Diffusion Equations via the Temimi-Ansari Method

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INFORMATION

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ABSTRACT

In this paper, a new analytical iterative method is used to obtain the fractional analytical solutions of the nonlinear gas dynamics and convection-diffusion equations. The paper's novelty appears in its specific application of the Caputo fractional operator to conventional equations while achieving highly accurate solutions. Numerical outcomes for various cases of the equations are represented via tables and graphs. The convergence analysis for the present approach was completed. The methodology is very capable of reducing the size of the analytical steps and is convenient and efficient for solving nonlinear fractional equations. The Temimi-Ansari method's applicability across different types of fractional differential equations indicates its potential as a powerful tool in solving nonlinear fractional models in diverse scientific and technical fields.

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1 Introduction

In the past decade, fractional calculus has seen applications in numerous scientific and technical fields [1,2]. Fractional differential equations (FDEs) have been used to study many linear and nonlinear real-life issues in nature [3–5]. They have been used increasingly to model problems in signal processing, propagation, acoustics, electromagnetism, biology, fluid mechanics, and many other physical processes [6–9]. It is a great tool for characterizing memory and inherited characteristics of different materials and processes. The value of FDE is that it has a non-local characteristic that exposes the new properties of these problems [10–12]. Even then, the exact solution to nonlinear FDEs is very difficult to obtain. It is clear that the burden, in general, arises from the nonlinearity of the fractional differential equation. We realize that many nonlinear differential equations cannot be easily solved. But with the presence of this number of computational methods that try hard to obtain approximate solutions, it became possible. Many of these techniques have been researched and narrated in great depth [13–17]. If we

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look at the current problem in detail, the Temimi-Ansari method (TAM) is a highly competitive technique in this field [18], for solving linear and nonlinear differential equations. This technique was proposed by Temimi and Ansari in 2011 and has been compared to different analytical techniques in other studies but in its classical case [19,20]. This study aims to solve the nonlinear gas dynamics and convection-diffusion equations using this effective computational approach. Compared to other existing techniques. We will use this technique to achieve accurate results to obtain quick convergence of the solution. The classical gas dynamic takes the form:

$$\frac{\partial\theta(\alpha,\beta)}{\partial\beta}+\frac{1}{2}\frac{\partial}{\partial\alpha}(\theta(\alpha,\beta))^2-\theta(\alpha,\beta)+(\theta(\alpha,\beta))^2=0,\alpha\in\mathbb{I},\beta>0. \tag{1}$$

The mathematical expressions of gas dynamics are predicated on physical laws of conservation, such as the conservation of momentum, laws of conservation of mass, conservation of energy, etc. Ideal gas dynamics equations apply to three types of nonlinear waves such as contact disconnection, shock interfaces, and dislocation. Several types of gas dynamics equations have been researched by using various numerical and analytical methods in physics [21–24]. Therefore, it has excellent significance in many fields of applied sciences and engineering. The discipline of mathematics that studies mass and heat transport is similar, and it typically takes diffusion and delay effects into account [25]. The classical convection-diffusion equations take the form:

$$\frac{\partial\theta(\alpha,\beta)}{\partial\beta}=\frac{\partial^2\theta(\alpha,\beta)}{\partial\alpha^2}-\frac{\partial\theta(\alpha,\beta)}{\partial\alpha}+\theta(\alpha,\beta)\frac{\partial\theta(\alpha,\beta)}{\partial\alpha}-(\theta(\alpha,\beta))^2+\theta(\alpha,\beta),\alpha\in\mathbb{I},\beta>0. \tag{2}$$

The convection-diffusion equations are used in many fields, such as sediment transport in rivers, estuaries, and coastal seas, water transport, flow in porous media, chemical absorption in beds, dispersion of reagents, flood propagation, salt penetration of water into fresh aquifers, heat transfer in discharge film, dispersion of dissolved materials in groundwater, dispersion of pollutants into rivers and streams, and dispersion of dissolved salts into groundwater [26–29]. These general scalar transport equations are commonly described as equations of convection-diffusion. This is one of the main nonlinear partial differential equations in which a fluid flow is necessary.

2 Preliminaries to Fractional Calculus

In this section, we will provide basic definitions of fractional calculus such as Riemann-Liouville (R-L) partial integration and Caputo fractional derivative. Caputo's concept of fractional differentiation will be used for this article. The Caputo sense has the benefit that the traditional form of the initial condition of the fractional partial differential equation (PDE) with the Caputo derivative is used.

2.1 Definition 1

The Caputo fractional operator is defined as [1]:

$$\mathfrak{J}^{m-\mu}\mathfrak{D}^{m}\theta(\beta)=\mathfrak{D}_*^{\mu}\theta(\beta)=\begin{cases}\dfrac{d^m}{d\beta^m}\theta(\beta), & \mu=m\in\mathbb{N},\\ \dfrac{1}{\alpha(m-\mu)}\int_{\varepsilon}^{\beta}(\beta-\varepsilon)^{m-\mu-1}\theta^{(m)}(\varepsilon)d\varepsilon, & m-1<\mu\leq m\in\mathbb{N}.\end{cases} \tag{3}$$

2.2 Definition 2

The Mittag-Leffler function $E_\mu(\beta)$ with $\mu > 0$ is defined as [27]:

$$E_\mu(\beta) = \sum_{j=0}^{\infty} \frac{\beta^j}{\alpha(j\mu + 1)}. \tag{4}$$

3 Construction of TAM of Fractional Order

We assume the generic fractional differential equation (FPDE) as follows to describe the main idea of the proposed technique [18–20]:

$$\Upsilon\left(\theta\left(\alpha, \beta\right)\right) + \Theta\left(\theta\left(\alpha, \beta\right)\right) = p\left(\alpha, \beta\right), m - 1 < \mu \leq m, \tag{5}$$

with the initial condition.

$$\mathfrak{B}\left(\theta, \frac{\partial \theta}{\partial \alpha}\right) = 0, \tag{6}$$

where $\Upsilon = \mathfrak{D}_\beta^\mu = \frac{\partial^\mu}{\partial \beta^\mu}$ is the Caputo fractional derivative, Θ is the generic differential operators, $\theta(\alpha, \beta)$ is representing the nameless function, the independent variable is denoted by and, the recognized continuous functions are represented by and the boundary operator is indicated by is the main requirement here and it is the general fractional differentiation, but we can take distinct linear expressions and lay them as needed along with the nonlinear expressions.

The suggested methodology begins with obtaining the initial condition as a result of eliminating the nonlinear part as:

$$\mathfrak{D}_\beta^\mu \theta_0(\alpha, \beta) = p(\alpha, \beta), \mathcal{B}\left(\theta_0, \frac{\partial \theta_0}{\partial \alpha}\right) = 0. \tag{7}$$

The next repetition of the answer is obtained by solving the following equation:

$$\mathfrak{D}_\beta^\mu \theta_1(\alpha, \beta) + \Theta\left(\theta_0(\alpha, \beta)\right) = p(\alpha, \beta), \mathcal{B}\left(\theta_1, \frac{\partial \theta_1}{\partial \alpha}\right) = 0. \tag{8}$$

As a result, we have a simple iterative stride $\theta_{m+1}(\alpha, \beta)$ which is the adequate approach to a linear and nonlinear set of problems.

$$\mathfrak{D}_\beta^\mu \theta_{m+1}(\alpha, \beta) + \Theta\left(\theta_m(\alpha, \beta)\right) = p(\alpha, \beta), \mathfrak{B}\left(\theta_{m+1}, \frac{\partial \theta_{m+1}}{\partial \alpha}\right) = 0. \tag{9}$$

In this approach, it is very important to note that either $\theta_{m+1}(\alpha, \beta)$ is solving for problem (5) separately. The iterative approach is easy to apply and each iteration improves upon the one before it. The iterative approach is easy to apply and each iteration is closer to the exact solution than the prior repetition. Continuing with this method, an ideal approximate solution corresponding to the exact solution can be obtained. In this way, the solution of Eq. (5) displayed as:

$$\theta(\alpha, \beta) = \lim_{m \to \infty} \theta_m(\alpha, \beta). \tag{10}$$

4 Applications on Semi-Analytical Algorithm

The TAM methodology was used to find analytical solutions for four applications to prove how good our algorithm was. These applications are gas dynamics equation (GD), and nonlinear convection-diffusion equation with fractional orders. Analytical and numerical analyses were detailed using the MATHEMATICA 12 software package during study time.

4.1 Application 1

The ensuing nonlinear fractional GD equation is seen as [21]:

$$\frac{\partial^{\mu}\theta(\alpha,\beta)}{\partial\beta^{\mu}}+\frac{1}{2}\frac{\partial}{\partial\alpha}(\theta(\alpha,\beta))^{2}-\theta(\alpha,\beta)+(\theta(\alpha,\beta))^{2}=0,0<\mu\leq1,\alpha\in\mathbb{I},\beta>0, \tag{11}$$

with initial condition.

$$\theta\left(\alpha,0\right)=e^{-\alpha}. \tag{12}$$

Through applying the analytical approach (TAM) of fractional order by firstly rewriting the equation as:

$$\Upsilon(\theta(\alpha,\beta))=\mathfrak{D}_{\beta}^{\mu}\theta(\alpha,\beta)=\frac{\partial^{\mu}\theta\left(\alpha,\beta\right)}{\partial\beta^{\mu}},\Theta(\theta(\alpha,\beta))=\theta(\alpha,\beta)\frac{\partial\theta\left(\alpha,\beta\right)}{\partial\alpha}-\theta(\alpha,\beta)$$
$$+(\theta(\alpha,\beta))^{2},p(\alpha,\beta)=0. \tag{13}$$

The initial condition considered as:

$$\Upsilon\left(\theta_{0}(\alpha,\beta)\right)=0,\theta_{0}\left(\alpha,0\right)=e^{-\alpha}. \tag{14}$$

We can solve Eq. (14) by utilizing a simple manipulation as follows:

$$\mathfrak{I}^{\mu}\left(\mathfrak{D}_{\beta}^{\mu}\theta_{0}(\alpha,\beta)\right)=0,\theta_{0}\left(\alpha,0\right)=e^{-\alpha}, \tag{15}$$

then, the main iteration is obtained from the fundamental properties of definition (2).

$$\theta_{0}\left(\alpha,\beta\right)=e^{-\alpha}. \tag{16}$$

We can compute the second iteration as:

$$\Upsilon\left(\theta_{1}(\alpha,\beta)\right)+\Theta\left(\theta_{0}(\alpha,\beta)\right)+p\left(\alpha,\beta\right)=0,\theta_{1}\left(\alpha,0\right)=e^{-\alpha}, \tag{17}$$

through the fundamental properties of the definition (2) and through the integration of both sides of the previous equation, we get:

$$\mathfrak{I}^{\mu}\left(\mathfrak{D}_{\beta}^{\mu}\theta_{1}\left(\alpha,\beta\right)\right)=-\mathfrak{I}^{\mu}\left(\theta_{0}\left(\alpha,\beta\right)\frac{\partial\theta_{0}\left(\alpha,\beta\right)}{\partial\alpha}-\theta_{0}\left(\alpha,\beta\right)+\left(\theta_{0}\left(\alpha,\beta\right)\right)^{2}\right),\theta_{1}\left(\alpha,0\right)=e^{-\alpha}. \tag{18}$$

Then, we get the subsequent iteration as:

$$\theta_{1}\left(\alpha,\beta\right)=e^{-\alpha}+\frac{e^{-\alpha}\beta^{\mu}}{\Gamma\left(\mu+1\right)}. \tag{19}$$

We can compute the third iteration as:

$$\Upsilon\left(\theta_{2}(\alpha,\beta)\right)+\Theta\left(\theta_{1}(\alpha,\beta)\right)+p\left(\alpha,\beta\right)=0,\theta_{2}\left(\alpha,0\right)=e^{-\alpha}, \tag{20}$$

through the fundamental properties of the definition (2) and through the integration of both sides of the previous equation, we get:

$$\mathfrak{I}^{\mu}\left(\mathfrak{D}_{\beta}^{\mu}\theta_{2}(\alpha,\beta)\right)=-\mathfrak{I}^{\mu}\left(\theta_{1}\left(\alpha,\beta\right)\frac{\partial\theta_{1}\left(\alpha,\beta\right)}{\partial\alpha}-\theta_{1}\left(\alpha,\beta\right)+\left(\theta_{1}\left(\alpha,\beta\right)\right)^{2}\right),\theta_{2}\left(\alpha,0\right)=e^{-\alpha}, \tag{21}$$

we get the third iteration as:

$$\theta_2(\alpha,\beta) = e^{-\alpha} + \frac{e^{-\alpha}\beta^{\mu}}{\Gamma(\mu+1)} + \frac{e^{-\alpha}\beta^{2\mu}}{\Gamma(2\mu+1)}. \tag{22}$$

The fourth iteration is:

$$\mathfrak{J}^{\mu}\left(\mathfrak{D}^{\mu}_{\beta}\theta_3(\alpha,\beta)\right) = -\mathfrak{J}^{\mu}\left(\theta_2(\alpha,\beta)\frac{\partial\theta_2(\alpha,\beta)}{\partial\alpha} - \theta_2(\alpha,\beta) + (\theta_2(\alpha,\beta))^2\right), \theta_3(\alpha,0) = e^{-\alpha}, \tag{23}$$

then, we get:

$$\theta_3(\alpha,\beta) = e^{-\alpha} + \frac{e^{-\alpha}\beta^{\mu}}{\Gamma(\mu+1)} + \frac{e^{-\alpha}\beta^{2\mu}}{\Gamma(2\mu+1)} + \frac{e^{-\alpha}\beta^{3\mu}}{\Gamma(3\mu+1)}. \tag{24}$$

Every iteration under Eq. (10) represents an approximation of the solution to Eq. (11). The approximate solution techniques the precise solution more closely as the number of iterations rises. The next approximate solution in a series form is obtained by continuing this procedure:

$$\begin{aligned}\theta(\alpha,\beta) &= \lim_{m\to\infty}\theta_m(\alpha,\beta)\\ &= e^{-\alpha}\left(1 + \frac{\beta^{\mu}}{\Gamma(\mu+1)} + \frac{\beta^{2\mu}}{\Gamma(2\mu+1)} + \frac{\beta^{3\mu}}{\Gamma(3\mu+1)} + \frac{\beta^{4\mu}}{\Gamma(4\mu+1)} + \cdots + \frac{\beta^{m\mu}}{\Gamma(m\mu+1)}\right)\\ &= e^{-\alpha}\sum_{j=0}^{n}\frac{(\beta^{\mu})^j}{\Gamma(j\mu+1)},\end{aligned} \tag{25}$$

which has the exact solution:

$$\theta(\alpha,\beta) = e^{-\alpha}E_{\mu}(\beta^{\mu}). \tag{26}$$

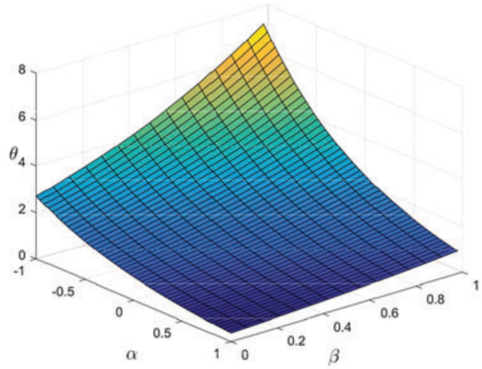
where E_{μ} is the Mittag-Leffler function [27]. If $\mu = 1$, we get the previous analytical solution as:

$$\theta(\alpha,\beta) = e^{-\alpha}\left(1 + \beta + \frac{\beta^2}{2!} + \frac{\beta^3}{3!} + \frac{\beta^4}{4!} + \frac{\beta^5}{5!} + \cdots\right). \tag{27}$$

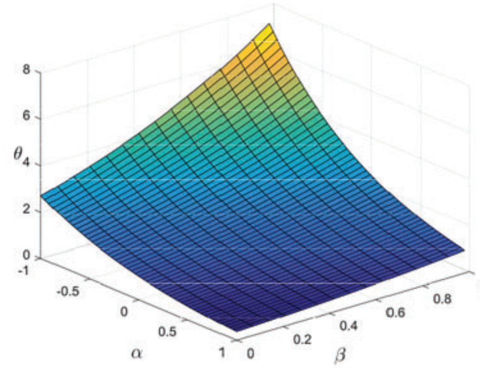
The previous analytical solution at $\mu = 1$ converges to the next exact solution [12].

$$\theta(\alpha,\beta) = e^{\beta-\alpha}. \tag{28}$$

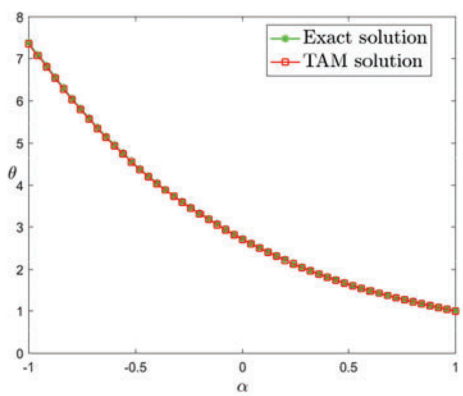
The approximate outcomes for the specific state $\mu = 1$ show that the analytical solution of Eq. (11) has the general technique which is identical with the precise solution (28). To show the approximate solution of Eq. (11) behaves geometrically, the precise solution was contrasted to the fifth repetition of the approximate solution, as shown in Fig. 1. The fifth iteration, uses $\mu = 1$, $\mu = 0.95$, $\mu = 0.90$ and $\mu = 0.80$, as compared to the precise solution. Ultimately, the second, third, fourth, and fifth iterations were compared with the precise result. Fig. 1 makes it clear that the behavior of each subfigure is the same and comparable. Furthermore, we observe that the Fig. 1c–e depicting fractional solutions are identical to one another and perfectly match the precise solution in terms of precision. When $m \to \infty$, we observe in Fig. 1f that the power series and Eq. (10) converge to the precise solution. Table 1 illustrates the quantitative outcomes of the proposed technique with different values for μ with changing space and time values.



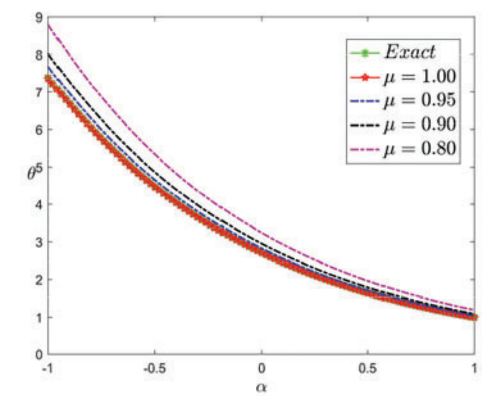
(a) Exact solution visualization.



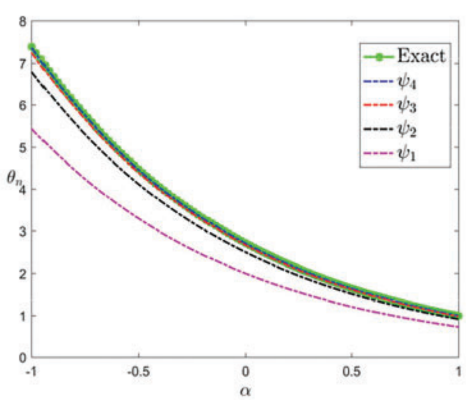
(b) FTAM approximate solution visualization.



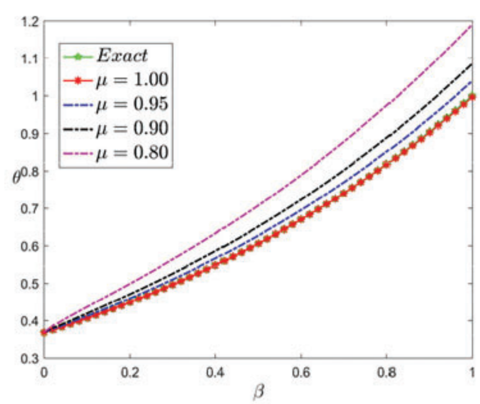
(c) Representation of the TAM and exact solutions.



(d) Representation of several amounts of μ at β = 1.



(e) The power series solution.



(f) Representation of several amounts of μ at α = 1.

Figure 1: Behavior of 5th order TAM for $\theta(\alpha, \beta)$ of Eq. (11) versus space α and time β

Table 1: Numerical solutions obtained by TAM of fractional order with different values of α, β and μ

		Gas dynamics equation			Convection-diffusion equation		
β	α	$\mu = 1$	$\mu = 0.9$	$\mu = 0.7$	$\mu = 1$	$\mu = 0.9$	$\mu = 0.7$
0.50	0	1.648719	1.772490	2.128519	1.648697	1.772391	2.126849
	2	0.223129	0.239880	0.288063	12.18232	13.09630	15.71540
	4	0.030197	0.032464	0.038985	90.01585	96.76931	116.1220
	6	0.004086	0.004393	0.005276	665.1322	715.0339	858.0322
	8	0.000553	0.000594	0.000714	4914.699	5283.425	6340.048
	10	0.000074	0.000080	0.000096	36314.98	39039.52	46846.97
0.75	0	2.116970	2.300881	2.822437	2.116723	2.300003	2.813268
	2	0.286500	0.311390	0.381975	15.64058	16.9948	20.78739
	4	0.038773	0.042142	0.051694	115.5691	125.5759	153.599
	6	0.005247	0.005703	0.006996	853.9472	927.8877	1134.953
	8	0.000710	0.000771	0.000946	6309.864	6856.214	8386.235
	10	0.000096	0.000104	0.000128	46623.94	50660.95	61966.36
1.00	0	2.718055	2.973997	3.690325	2.716666	2.969844	3.659630
	2	0.367848	0.402486	0.499431	20.07360	21.9443	27.0412
	4	0.049782	0.054470	0.067590	148.3249	162.1480	199.8090
	6	0.006737	0.007371	0.009147	1095.981	1198.120	1476.400
	8	0.000911	0.000997	0.001237	8098.269	8852.982	10909.20
	10	0.000123	0.000135	0.000167	59838.56	65415.187	80608.71

4.2 Application 2

The ensuing nonlinear fractional convection-diffusion equation is seen as [30]:

$$\frac{\partial^{\mu}\theta(\alpha,\beta)}{\partial\beta^{\mu}} = \frac{\partial^{2}\theta(\alpha,\beta)}{\partial\alpha^{2}} - \frac{\partial\theta\,(\alpha,\beta)}{\partial\alpha} + \theta(\alpha,\beta)\frac{\partial\theta\,(\alpha,\beta)}{\partial\alpha} - (\theta(\alpha,\beta))^{2}$$
$$+\,\theta(\alpha,\beta), 0 < \mu \le 1, \alpha \in \mathbb{I}, \beta > 0, \tag{29}$$

with initial condition:

$$\theta\,(\alpha,0) = e^{\alpha}. \tag{30}$$

Stratifying the same basal concept of the TAM of fractional order, we get the next analytical solutions:

$$\left.\begin{array}{r}\theta_0(\alpha,\beta)=e^{\alpha}\\ \theta_1(\alpha,\beta)=e^{\alpha}+\dfrac{e^{\alpha}\beta^{\mu}}{\Gamma(\mu+1)},\\ \theta_2(\alpha,\beta)=e^{\alpha}+\dfrac{e^{\alpha}\beta^{\mu}}{\Gamma(\mu+1)}+\dfrac{e^{\alpha}\beta^{2\mu}}{\Gamma(2\mu+1)},\\ \theta_3(\alpha,\beta)=e^{\alpha}+\dfrac{e^{\alpha}\beta^{\mu}}{\Gamma(\mu+1)}+\dfrac{e^{\alpha}\beta^{2\mu}}{\Gamma(2\mu+1)}+\dfrac{e^{\alpha}\beta^{3\mu}}{\Gamma(3\mu+1)},\cdots\end{array}\right\}. \tag{31}$$

Every iteration under Eq. (10) represents an approximation of the solution to Eq. (29). The approximate solution techniques the precise solution more closely as the number of iterations rises. The next approximate solution in a series form is obtained by continuing this procedure:

$$\begin{aligned}\theta(\alpha,\beta)&=\lim_{m\to\infty}\theta_m(\alpha,\beta)\\ &=e^{\alpha}\left(1+\frac{\beta^{\mu}}{\Gamma(\mu+1)}+\frac{\beta^{2\mu}}{\Gamma(2\mu+1)}+\frac{\beta^{3\mu}}{\Gamma(3\mu+1)}+\frac{\beta^{4\mu}}{\Gamma(4\mu+1)}+\cdots+\frac{\beta^{m\mu}}{\Gamma(m\mu+1)}\right)\\ &=e^{\alpha}\sum_{j=0}\frac{(\beta^{\mu})^{j}}{\Gamma(j\mu+1)}.\end{aligned} \tag{32}$$

which has the exact solution:

$$\theta(\alpha,\beta)=e^{\alpha}E_{\mu}(\beta^{\mu}). \tag{33}$$

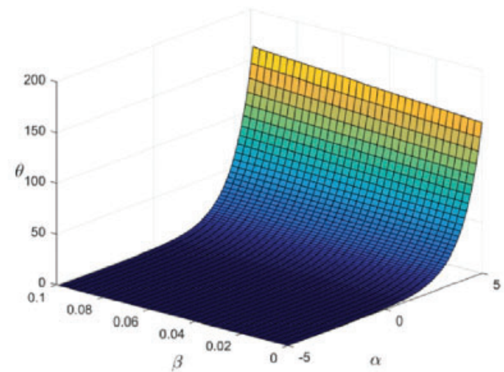
If $\mu=1$, we get the previous analytical solution as:

$$\theta(\alpha,\beta)=e^{\alpha}\left(1+\beta+\frac{\beta^2}{2!}+\frac{\beta^3}{3!}+\frac{\beta^4}{4!}+\frac{\beta^5}{5!}+\cdots\right). \tag{34}$$

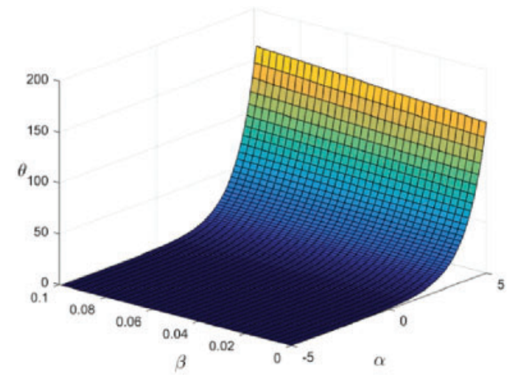
The previous analytical solution at $\mu=1$ converges to the next exact solution [28]:

$$\theta(\alpha,\beta)=e^{\alpha+\beta}. \tag{35}$$

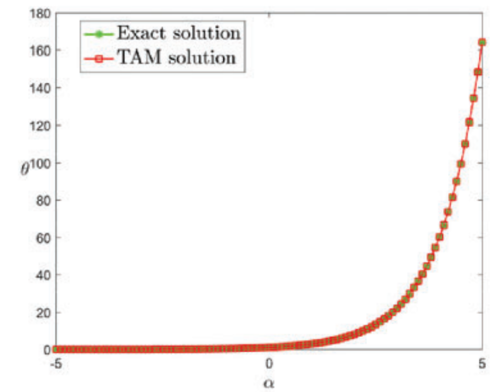
The approximate outcomes for the specific state $\mu=1$ show that the analytical solution of Eq. (29) has the general technique which is identical with the precise solution (35). To show the approximate solution of Eq. (29) behaves geometrically, the precise solution was contrasted to the fifth repetition of the approximate solution, as shown in Fig. 2. The fifth iteration, uses $\mu=1$, $\mu=0.95$, $\mu=0.90$ and $\mu=0.80$, as compared to the precise solution. Ultimately, the second, third, fourth, and fifth iterations were compared with the precise result. Fig. 2 makes it clear that the behavior of each sub-figure is the same and comparable. Furthermore, we observe that the Fig. 2c–e depicting fractional solutions are identical to one another and perfectly match the precise solution in terms of precision. When $m\to\infty$, we observe in Fig. 2f that the power series and Eq. (10) converge to the precise solution. Table 1 illustrates the quantitative outcomes of the proposed technique with different values for μ with changing space and time values.



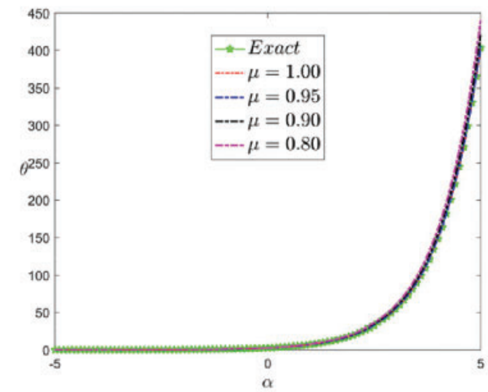
(a) Exact solution visualization.



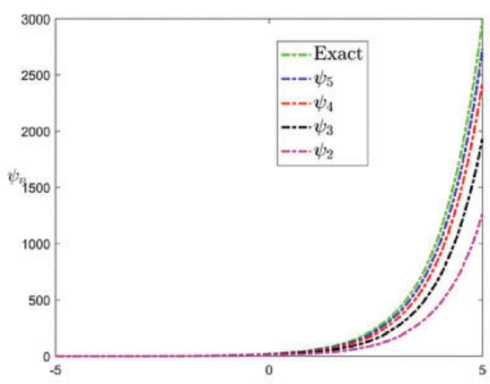
(b) FTAM approximate solution visualization.



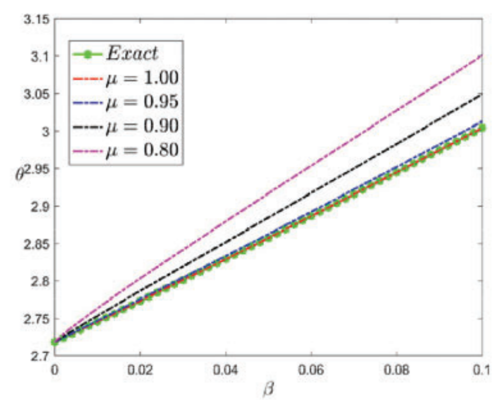
(c) Representation of the TAM and exact solutions.



(d) Representation of several amounts of μ at β = 1.



(e) The power series solution.



(f) Representation of several amounts of μ at α = 1.

Figure 2: Behavior of 6th order TAM for $\theta(\alpha, \beta)$ of Eq. (29) versus space α and time β

5 Conclusion

In this work, approximate solutions for nonlinear fractional gas dynamics and convection-diffusion equations have been obtained by the successful use of the TAM. The method handles nonlinear fractional differential equations efficiently and simply utilizing the Caputo fractional operator to get the correct analytical solutions. Numerical data collected from a variety of cases and displayed in tables and graphs verified the method's convergence and reliability. The results highlight the usefulness of TAM in cutting down on computing steps without sacrificing accuracy, which is especially beneficial for complicated fractional equations in the applied sciences. This method's versatility and accuracy position it as a powerful tool for researchers in fields such as fluid dynamics, environmental science, and engineering, where fractional dynamics play a significant role. Future research could extend this method to other fractional differential equations, further establishing TAM's utility in modeling and solving real-world problems.

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Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

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