

## ROCKING ANALYSIS FOR THE BELL TOWER OF SANT'ANNA IN CERVINO

**Antonio Gesualdo<sup>1</sup>, Anna Tafuro<sup>2</sup>, Mariateresa Guadagnuolo<sup>2</sup>**

<sup>1</sup> Department of Structures for Engineering and Architecture, University of Naples “Federico II”, Via Claudio 21, 80125 Napoli, Italy, gesualdo@unina.it

<sup>2</sup> Department of Architecture and Industrial Design, University of Campania “Luigi Vanvitelli”, Via San Lorenzo, 81031 Aversa (CE), Italy, {anna.tafuro, mariateresa.guadagnuolo}@unicampania.it

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**Abstract.** In seismic prone areas ecclesiastical masonry complexes have shown a very high vulnerability, as detected after the last Italian earthquakes, such as those occurred in L'Aquila (2009), Emilia-Romagna (2012), Central Italy (2016), and Ischia (2017). These are particular types of aggregate buildings subjected often to partial collapses, due to the presence of highly vulnerable elements, like the bell towers. Preliminary analyses should including straightforward and quick methods are necessary. In this paper the bell tower vulnerability is analyzed taking into account the rocking behaviour of the tower only and considering the contribute of the entire ecclesiastical complex as a rigid body sliding with a fixed friction coefficient with respect to the foundations. It is shown that suitable values of maximum oscillations and horizontal displacements are obtained. The case study is the ecclesiastical complex of S. Anna in Cervino (Caserta, Italy).

### 1 INTRODUCTION

Early studies on the rocking response of a rigid block supported on a base undergoing horizontal motion were presented by Housner [1], who first established the equations of motion of the rigid body and solved them accordingly. The study was devoted to the understanding of the behaviour of tall, slender structures subjected to ground motion. Only recently literature papers address different behaviours that can be recognized and modelled according the Housner theory, such as the art objects [2, 3], non structural elements [4, 5], rocking of bell towers due to the bells swinging [6, 7].

In general, the bell towers are slender elements whose structure is part of larger masonry buildings whose behaviour should be properly assessed considering the entire complex [8, 9, 10], especially when retrofit interventions are planned [11, 12]. In recent times the strong need of rehabilitation, together with the development of numerical tools, has improved knowledge about assessment methods for masonry buildings [13], but the work is strongly complex, due to the presence of substructures like arches and vaults [14, 15, 16, 17], walls in out of plane [18] or in-plane behaviour [19, 20], single columns [21]. The modelling problem is often solved examining the behaviour of the single substructure extracted from the more complex masonry building. In this paper the bell tower vulnerability is analyzed taking into account the rocking behaviour of the tower only and considering the contribute of the entire ecclesiastical complex

as a rigid body sliding with a fixed friction coefficient with respect to the foundations. It is shown that suitable values of maximum oscillations and horizontal displacements are obtained. The case study is the ecclesiastical complex of S. Anna in Cervino (Caserta, Italy), where the tower, whose behaviour is modelled according the Housner's theory of inverted pendulum, is a sort of slender element uprising from the bulk masonry building, able to rock on its base, as shown in the following picture.

Housner's model is so that examined and applied to the dynamics of the bell tower of Sant'Anna in Cervino, considered as a rigid block. The oscillation of the bell tower only with respect to the base edges parallel to the minimum plane of inertia is considered.

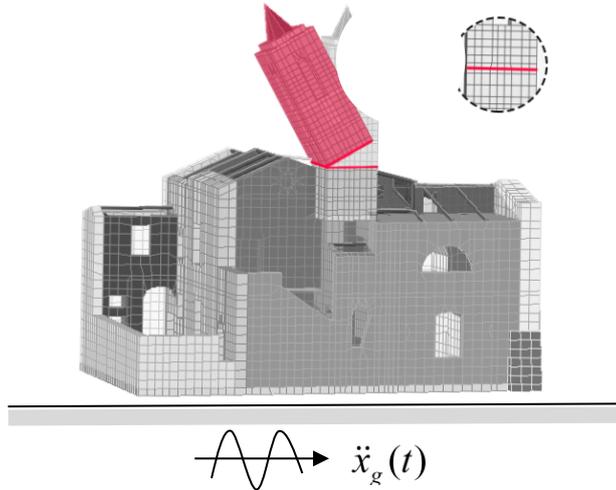


Figure 1. Rigid block behaviour of the bell tower

Following Housner's theory of the inverse pendulum [1963], the bell tower is considered as a single rigid block, simply resting on a horizontal plane with friction, belonging to the entire building, which in turn is able to slide with friction with respect to the laying surface of the foundations, this last subject to horizontal acceleration. The model is more complex than that proposed by Housner in 1963, since it combines two distinct motions: the Housner-like oscillation of the bell tower alone and the frictional translation of the underlying building. In order to evaluate the overall model, the two distinct simple motions are examined and then the combination of the two motions with the possible evolutions for the dynamics of the bell tower.

## 2 THE INVERTED PENDULUM

### 2.1 Rocking

The Housner model, developed in the early 1970s, considers a symmetrical rigid block, with base  $B$  and height  $H$  simply resting on a horizontal plane in oscillatory motion with acceleration  $\ddot{x}_g(t)$ . The rigid block (in the examined case the bell tower only) can rotate alternatively with respect to the two points  $O$  and  $O'$  of the base  $B$  with a rotation angle  $\theta$ , positive if clockwise. The impact with the base when the direction of rotation changes is the only dissipative event in the motion.

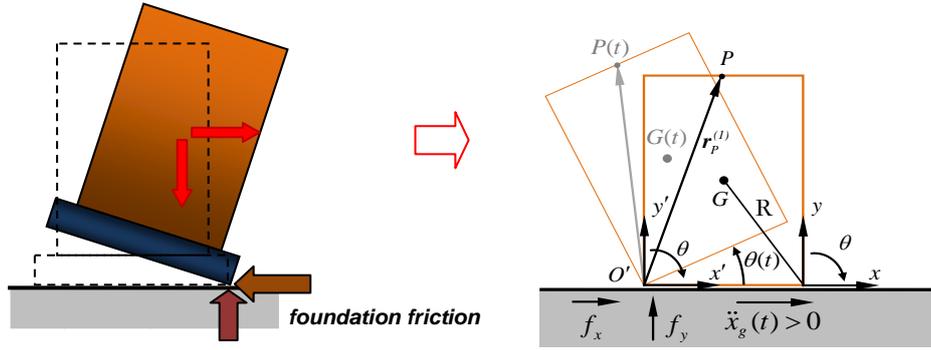


Figure 2. Rocking of a construction (left) and Housner model (right)

According to Housner's theory, the speed after a perfectly centered impact is related to the pre-impact speed through a reduction coefficient  $e$ , which is related to the restitution coefficient  $r$  defined by Housner through the relationship  $e = \sqrt{r}$ . It can be assumed that the reduction coefficient remains constant throughout the motion, i.e. that the amount of kinetic energy dissipated is always the same, so that the angular velocity of the block after the impact  $\dot{\theta}^+(t)$  maintains a constant relationship with the pre-impact one  $\dot{\theta}^-(t)$  (Figure 2):

$$\dot{\theta}^+(t) = r \dot{\theta}^-(t). \quad (1)$$

In these hypotheses the conservation of angular momentum about point  $O'$  just before the impact and right after the impact is:

$$(I_o - 2mr b \sin\alpha \dot{\theta}^-(t)) = I_o \dot{\theta}^+(t). \quad (2)$$

The value of  $r$  for a rectangular block can be derived by a combination of (1) and (2):

$$r = 1 - \frac{3}{2} \sin^2\alpha \quad \text{with} \quad 0 < r < 1. \quad (3)$$

When the rotation axis instantaneously moves from  $O$  to  $O'$  and conversely the coefficient of restitution is a measure of the energy lost during the impact. Rocking motion is present when the static friction with the base plane is so great as to prevent sliding. Adopting the notation by Shenton [1996], let  $f_x$  and  $f_z$  be the horizontal and vertical reactions at the tip  $O'$  of the block, at all times rocking motion holds true if:

$$|f_x| \leq \mu_s f_y. \quad (4)$$

In other words, starting from an equilibrium configuration of the system and given the condition (4), the angular momentum of inertia forces is greater than that due to gravity force.

The rocking motion, according to the D'Alembert principle, is governed by the following set of differential equations (DEs):

$$\begin{aligned}
I_{O'} \ddot{\theta}(t) + m g R \cdot \text{Sin}(-\alpha - \theta(t)) &= -m \ddot{x}_g(t) R \cos(-\alpha - \theta(t)) , \theta(t) < 0 \\
I_O \ddot{\theta}(t) + m g R \sin(\alpha - \theta(t)) &= -m \ddot{x}_g(t) R \cos(\alpha - \theta(t)) , \theta(t) > 0 \\
\dot{\theta}^+(t) &= r \dot{\theta}^-(t) , \theta(t) = 0
\end{aligned} \tag{5}$$

where  $\ddot{x}_g(t)$  is the horizontal base acceleration,  $I_O = I_{O'}$  is the polar inertia moment with respect to the two points  $O$  and  $O'$  and the rocking motion starts when  $|\ddot{x}_g(t)| > g b/h$ , being  $g$  the gravity acceleration. The first two ordinary nonlinear differential equations are relative to the rotation motion around  $O$  and  $O'$  and the third algebraic equation relates the two angular velocities in  $O$  and  $O'$  and holds true at the impact instant only. The angle  $\alpha = \arctan b/h$  takes into account the slenderness of the block. By the signum function:

$$\text{sgn}(\theta(t)) = \begin{cases} +1 & \theta(t) > 0 \\ -1 & \theta(t) < 0 \end{cases}$$

the system (5) can assume the following form:

$$\begin{aligned}
\frac{I_o}{mR} \ddot{\theta}(t) + g \text{sgn}(\theta(t)) \sin(\alpha - \text{sgn}(\theta(t))\theta(t)) &= -\ddot{x}_g(t) \cos(\alpha - \text{sgn}(\theta(t))\theta(t)) , \theta(t) \neq 0 \\
\dot{\theta}^+(t) &= r \dot{\theta}^-(t) , \theta(t) = 0.
\end{aligned} \tag{6}$$

The numerical solution of the DEs (6) may be put more conveniently in terms of a key point displacement, considering two reference systems with origin in the two rotation points  $O$  and  $O'$ , namely  $\mathcal{R}_1 = \{O, x, y\}$  for  $\theta(t) > 0$  and  $\mathcal{R}_2 = \{O', x', y'\}$  for  $\theta(t) < 0$ .

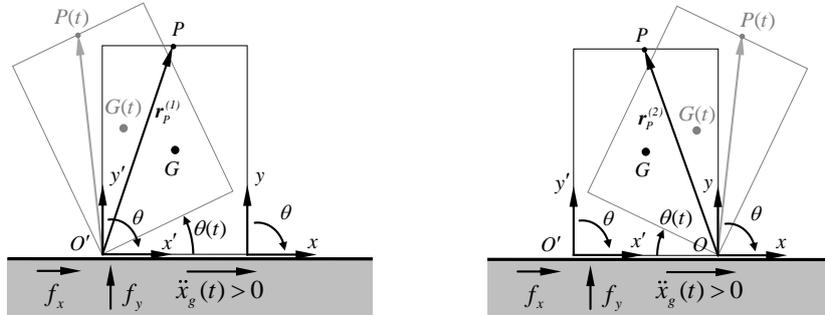


Fig. 2. Reference systems for the single rocking block in rocking for  $\theta(t) < 0$  (left) and  $\theta(t) > 0$  (right)

Let  $\theta(t)$  be the rotation function, the position of the point  $P$  at time  $t$  in the two frame systems above described is related to the position vector at the starting time:

$$\mathbf{r}_P^{(1)} = \begin{bmatrix} x_P^{(1)} \\ y_P^{(1)} \end{bmatrix} , \quad \theta(t) < 0 \quad ; \quad \mathbf{r}_P^{(2)} = \begin{bmatrix} x_P^{(2)} \\ y_P^{(2)} \end{bmatrix} , \quad \theta(t) > 0 \tag{7}$$

so that the actual position of the point  $P$  is given by the rotation matrix  $\mathbf{R} \circ \theta(t)$  applied on  $\mathbf{r}_P^{(1)}$  and  $\mathbf{r}_P^{(2)}$ :

$$\begin{aligned} OP(t) &= \mathbf{R} \circ \theta(t) \mathbf{r}_p^{(1)} , \theta(t) > 0 \\ O'P(t) &= \mathbf{R} \circ \theta(t) \mathbf{r}_p^{(2)} , \theta(t) < 0 \end{aligned} \quad (8)$$

where the rotation matrix  $\mathbf{R} \in SO(2)$ , being  $SO(2)$  the orthogonal group of matrices with  $\det(\mathbf{R}) = 1$ , is:

$$\mathbf{R} \circ (\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\ -\sin(\cdot) & \cos(\cdot) \end{bmatrix} \quad (9)$$

from (8) the acceleration is derived as:

$$\begin{aligned} \frac{\partial^2}{\partial t^2} OP(t) &= \frac{\partial^2}{\partial t^2} [\mathbf{R} \circ \theta(t)] \mathbf{r}_p^{(1)} , \theta(t) > 0 \\ \frac{\partial^2}{\partial t^2} O'P(t) &= \frac{\partial^2}{\partial t^2} [\mathbf{R} \circ \theta(t)] \mathbf{r}_p^{(2)} , \theta(t) < 0 \end{aligned} \quad (10)$$

after some manipulations the (10) can be rewritten as follows:

$$\begin{aligned} \frac{\partial^2}{\partial t^2} OP &= [\ddot{\theta}(t) \partial \mathbf{R} \circ \theta(t) - \dot{\theta}^2(t) \mathbf{R} \circ \theta(t)] \mathbf{r}_p^{(1)} , \theta(t) > 0 \\ \frac{\partial^2}{\partial t^2} O'P &= [\ddot{\theta}(t) \partial \mathbf{R} \circ \theta(t) - \dot{\theta}^2(t) \mathbf{R} \circ \theta(t)] \mathbf{r}_p^{(2)} , \theta(t) < 0 \end{aligned} \quad (11)$$

where the first derivative of the rotation matrix  $\mathbf{R}$  belongs to the orthogonal group of matrices with unit determinant:

$$\partial \mathbf{R} \circ (\cdot) = \begin{bmatrix} -\sin(\cdot) & \cos(\cdot) \\ -\cos(\cdot) & -\sin(\cdot) \end{bmatrix} \in SO(2).$$

The horizontal component of relative acceleration can be deduced by (11):

$$\ddot{\mathbf{x}}(t) = \begin{cases} \frac{\partial^2}{\partial t^2} OP \cdot \mathbf{i} , \theta(t) > 0 \\ \frac{\partial^2}{\partial t^2} O'P \cdot \mathbf{i} , \theta(t) < 0 \end{cases}$$

with  $\mathbf{i}$  unit vector of  $x$  axis. The horizontal acceleration  $\ddot{\mathbf{x}}(t)$  can be put in the explicit form:

$$\ddot{\mathbf{x}}(t) = \begin{cases} -[x_1 \cos(\theta(t)) + y_1 \sin(\theta(t))] \dot{\theta}(t) + [-x_1 \sin(\theta(t)) + y_1 \cos(\theta(t))] \ddot{\theta}(t) , \theta(t) > 0 \\ -[x_2 \cos(\theta(t)) + y_2 \sin(\theta(t))] \dot{\theta}(t) + [-x_2 \sin(\theta(t)) + y_2 \cos(\theta(t))] \ddot{\theta}(t) , \theta(t) < 0. \end{cases} \quad (12)$$

The absolute acceleration:

$$\ddot{\mathbf{x}}_a(t) = \ddot{\mathbf{x}}_g(t) + \ddot{\mathbf{x}}(t). \quad (13)$$

is the sum of the base acceleration and the block one.

## 2.2 Sliding

The configuration of the block in case of sliding motion can be characterized by the translation of a generic point of the block with respect to the base.

The friction force is function of the vertical forces applied on the block and is opposite to the motion. Starting from an equilibrium configuration, sliding motion begins when the maximum horizontal force due to the static friction coefficient is attained.

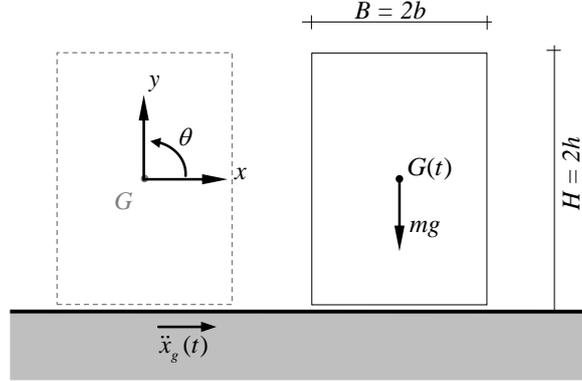


Fig. 3. The single sliding block

With reference to the scheme of Fig. 3, the governing equations are:

$$\begin{aligned} |\ddot{x}_g(t)| &> g \mu_s \\ (\ddot{x}_g(t) + \ddot{x}(t)) &= -\text{sgn}(\dot{x}(t)) g \mu_k. \end{aligned} \quad (14)$$

Starting from the instant in which the friction contact force is exceeded by the inertial forces related to (13), the differential equation of sliding (14<sub>2</sub>) is integrated in the numerical procedure until the relative velocity  $\dot{x}(t)$  is nonzero. When the velocity becomes null the block is in relative equilibrium with the base (rest) until the external force attains a value able to reactivate the sliding motion.

## 3 COMBINED MOTIONS BUILDING-BELL TOWER

The combined motion analysis was recently developed by [22] and the model assumes that the upper block (the bell tower) can only make oscillations with respect to the underlying building, due to the low tensile strength of mortar layer at the level of springing [23], which instead is able to translate without oscillations with respect to to the foundation plane, and therefore the problem is governed by the set of (5) and (8), i.e. in compact form:

$$\begin{cases} J_o \ddot{\theta}(t) - m_2 R \cos[\alpha - |\theta|] (\ddot{x}_g(t) + \ddot{x}_{G_1}(t)) + \\ + m_2 R g \text{sgn}(\theta(t)) \sin[\alpha - |\theta|] = 0 & , \theta(t) \neq 0 \\ M (\ddot{x}_g(t) + \ddot{x}_{G_1}(t)) + \text{sgn}(\theta(t)) \{-m_2 R [\sin(\alpha - |\theta|) \dot{\theta}^2(t) + \\ - \cos(\alpha - |\theta|) \ddot{\theta}(t)] + M \mu_k g\} = 0 & , \theta(t) \neq 0 \\ \dot{\theta}^+(t) = r \dot{\theta}^-(t) & , \theta(t) = 0 \end{cases} \quad (15)$$

where  $\text{sgn}(\theta(t))$  is the signum function taking into account the oscillations with respect the two base corners of the bell tower.

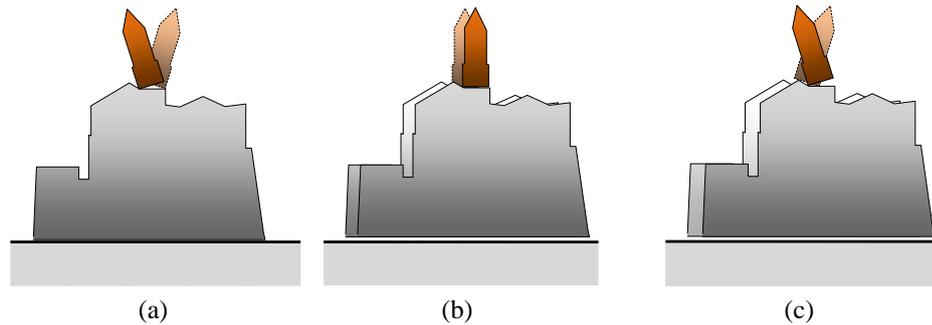


Figure 4. Possible motions

There are three possible combined motions:

- oscillation of the bell tower with the building perfectly jointed with the underlying foundation in motion
- sliding of the entire building, including the bell tower that does not swing, with respect to the foundation;
- combined motion: the bell tower oscillates with respect to the building that undergoes relative displacements with respect to the foundation plane.

#### 4 CASE STUDY

The case study is the ecclesiastical complex of S. Anna in Cervino, Caserta, reported in the following picture, where the church, the sacristy and the bell tower are indicated. These masonry aggregates are in fact key in the evaluation of vulnerability at building [24, 25, 26, 27] and urban scale [28]



Figure 5. The case study

Figure 6 reports the rocking analyses on the bell tower, considered in the double block

system as previously analyzed, varying friction coefficients, static ( $\mu_s = 1.25$ ) and kinematic ( $\mu_k = 1$ ), and the frequency in the interval  $[3, 7]$  Hz. As it can be noted, for increasing frequency, the amplitude of rotation decreases. In Figure 7  $\theta(t)$  diagrams varying amplitude of acceleration, frequencies, friction coefficients are shown.

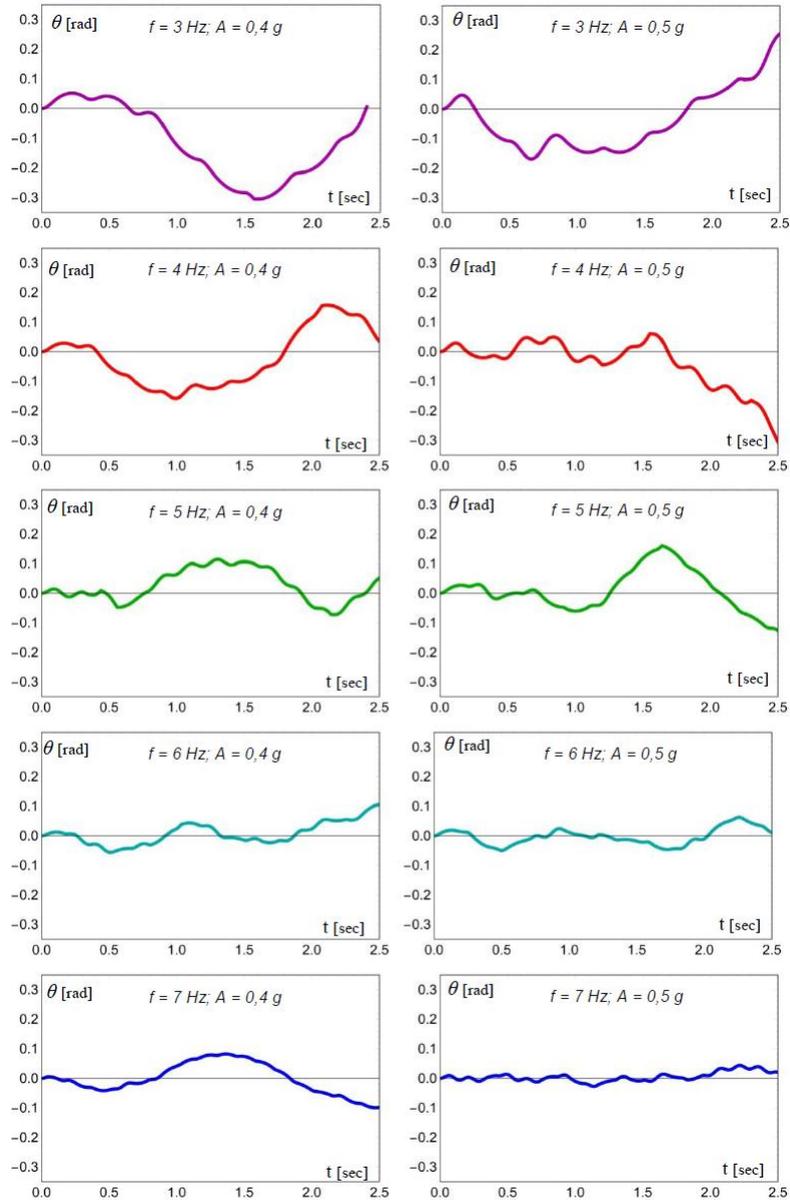


Figure 6. Friction coefficients: static  $\mu_s = 1.25$ , kinematic  $\mu_k = 1$

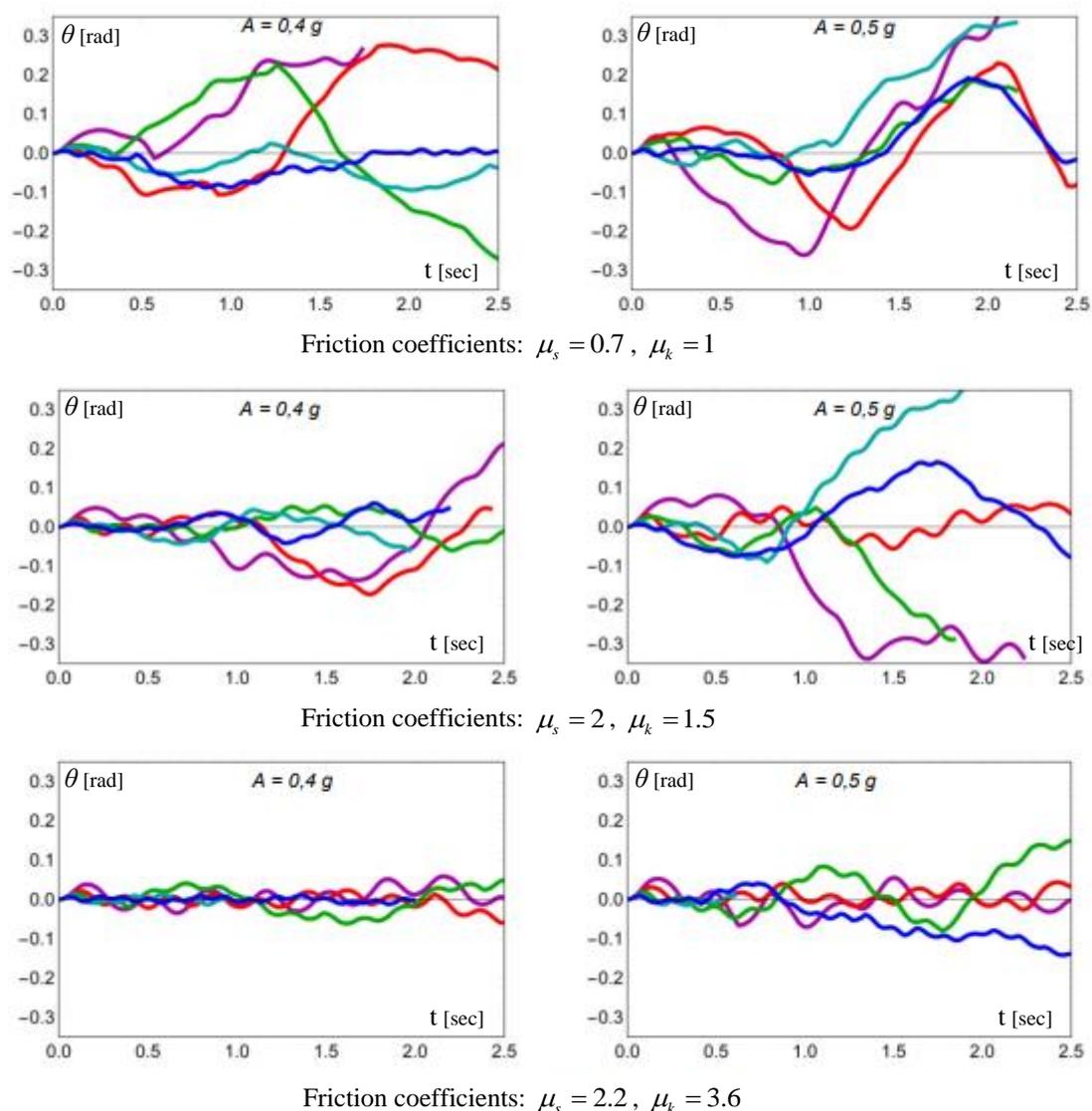


Figure 7.  $\theta(t)$  diagrams varying amplitude of acceleration, frequencies, friction coefficients

## 5 CONCLUSIONS

The vulnerability of the bell tower of S. Anna in Cervino (Caserta, Italy) has been analyzed considering the rocking behaviour of the tower. The effect of the global ecclesiastical complex as a rigid body sliding with a fixed friction coefficient with respect to the foundations has been taken into account in the rocking behaviour of the bell tower. Oscillation diagrams of the bell tower with several values of amplitude of the ground acceleration, of frequency and friction coefficient are presented. It is noted that high values of frequencies and coefficients of friction make the bell tower stable. The variation in stability, as the graphs show, is not strictly proportional to these parameters, as shown by the final diagrams.

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