IX International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2021
E. Oñate, M. Papadrakakis and B. Schrefler (Eds)

MECHANICAL CONSTRAINT ARRANGEMENT AND ITS MULTIBOND GRAPH REPRESENTATION

IVAN I. KOSENKO*

*Institute of Applied Mathematics and Physics
Moscow Aviation Institute (National Research University)
Volokolamskoe shosse 4, 125993, Moscow, Russia
e-mail: kosenkoii@gmail.com

Key words: Multibond Graph, Wrenches, Twists, Representation of Constraint, Object-Oriented Modeling, Junction Structure

Abstract. When developing a computer model of the multibody system (MBS) dynamics it is interesting to have a unified technology to construct the models in an efficient way. It turns out object-oriented approach provides a tools to resolve such a problem successively step by step. One of these unified ways is connected tightly with the so-called multiport representation of the models initially based on the bond graph use. These latter ones in turn based on the idea of energy exchanges, and substantially on energy conservation for physically interconnected subsystems of any engineering type.

A detailed description of the multibond graph representation for the most general type of constraint is presented. It turned out the resulting total multibond graph model of the multibody system dynamics always has exactly a canonical junction structure. This representation has a tight correspondence with our previous object-oriented implementation of the mechanical constraint architecture.

Computational experience makes it possible to classify objects of the multibody system dynamics. Such a model includes two classes of objects. They correspond to notions of “body” and “constraint”. Each of these notions indeed corresponds to the certain type of the multibond graph junction.

1 INTRODUCTION

When developing a computer model of the MBS it is interesting to have a unified technology to construct the models in an efficient way [1]. It turns out Modelica language provides a tools to resolve such a problem successively step by step using its natural approaches [2, 3]. One of them is connected tightly with the so-called multiport representation of the models initially based on the bond graph use [4]. These latter in turn based on the idea of energy interaction [5], and substantially on energy conservation for physically interconnected subsystems of any engineering type.

A lot of methods to describe the structure of the MBS using different graph approaches is known, see for instance [6]. Consider the MBS consisting of $m+1$ bodies $B_0, \ldots, B_m$. Represent it as a set $\mathcal{B} = \{B_0, \ldots, B_m\}$. Here B_0 is assumed to be a base body. We suppose B_0 to be connected with an inertial frame of reference, or to have a known motion with respect to the inertial frame of reference. For example one can imagine the base body as a rotating platform, or as a vehicle performing its motion according to a given law. For definiteness and simplicity we suppose in the sequel all state variables describing the rigid bodies motion always refer to one fixed inertial coordinate system connected to the base body by default.

Some bodies are considered as connected by mechanical constraints. Suppose all constraints compose the set $\mathcal{C} = \{C_1, \ldots, C_n\}$. We include in our considerations constraints of the following types: holonomic/nonholonomic, scleronomic/rheonomic.

Thus one can uniquely represent a structure of the MBS via a undirected graph $G = (\mathcal{B}, \mathcal{C}, \mathcal{I})$. Here $\mathcal{I} \subset \mathcal{C} \times \mathcal{B}$ is an incidence relation setting in a correspondence the vertex incident to every edge $C_i \in \mathcal{C}$ of the graph. According to physical reasons it is easy to see that for any mechanical constraint C_i there exist exactly two bodies $B_k, B_l \in \mathcal{B}$ connected by this constraint.

2 CONSTRAINT REPRESENTATION VIA BOND GRAPHS

Previously, when considering a unified model of the constraint, or, in a more general way, any physical interaction between two rigid/deformable bodies we defined [1, 7] two classes of the kinematic and the effort ports. These ones are the kinematic and wrench connectors. It turned out the connections of such types make it possible to construct a model of the bodies interactions based on the causality physically motivated.

Namely, the constraint object imports the kinematic information accepting it from the objects of interacting bodies and reciprocally exports it in the opposite direction. Thus the constraint "computes" an efforts the bodies interact by.

On the other hand geometric formalisms to represent the MBS dynamics are known [8, 15] which operates with the similar information objects: twists and wrenches. In our approach twist is defined by the `KinematicPort` class, and wrench obviously corresponds to our `WrenchPort` class. The representation under consideration is tightly connected with the power based approach to modeling, so-called bond graphs [9].

Indeed, let the rigid body kinematics be defined by the twist $(\mathbf{v}, \boldsymbol{\omega})$, where $\mathbf{v}$ is the mass center velocity, and $\boldsymbol{\omega}$ is the body angular velocity. Further let all the forces acting upon the body be reduced to the wrench $(\mathbf{F}, \mathbf{M})$ with the total force $\mathbf{F}$ and the total torque $\mathbf{M}$. Thus the total power of all the forces acting on the body is computed by the known formula

$$W = (\mathbf{v}, \mathbf{F}) + (\boldsymbol{\omega}, \mathbf{M})$$

using to represent a multibond in the bond graphs simulating the MBS dynamics. We have in such the case an evident canonical duality between twists and wrenches.

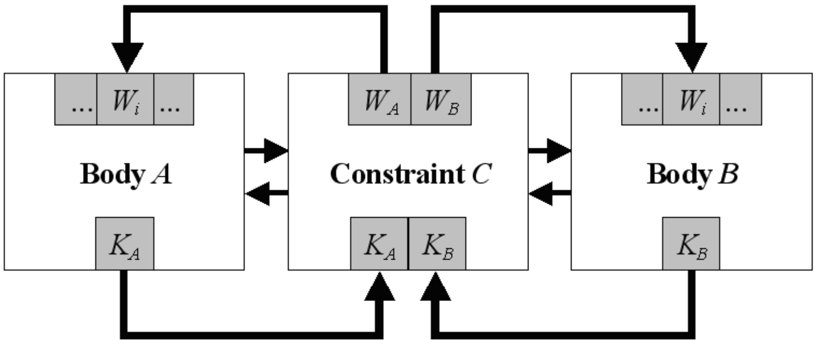

Figure 1: Architecture of Constraint

Sometimes wrenches are selected as flow variables. In other cases twists play this role. For instance similarities between electricity and mechanics cause the parallelism for electric current and forces/torques in one dimensional powertrains of mechanisms. In this case we can set a correspondence between the Kirchhoff law for currents and the d'Alembert principle for external forces and forces of inertia "acting" upon the body.

In our opinion it may be interesting enough to apply an approach dual to the first one mentioned above. Such an approach is more natural in traditional classical mechanics and assumes twist for the flow variable in the multibond. In the further course we present an illustration for this approach and demonstrate its convenience to construct the mechanical constraints of different types. Moreover, object-oriented implementation may be interpreted in both above dual approaches in a symmetric ways.

Let us trace now the similarities between the bond graphs and our MBS models. Evidently the pair of classes `KinematicPort/WrenchPort` plays a role of the multiport notion, and corresponding pairs of connections in Figure 1 stand for the notion of a bond.

Furthermore, in this way we can associate an object of the `RigidBody` class with 1-junction, while 0-junction is associated with the object of the class `Constraint`. The relevant general bond graph representation of the constraint in any MBS may be depicted as it shown in Figure 2.

All multibonds here consist of the twist $(\mathbf{v}, \boldsymbol{\omega})$ signals representing the flow component, and the wrench $(\mathbf{F}, \mathbf{M})$ signals as an effort. Causality of an inertance elements arranges according to the Newton–Euler system of ODEs. Left and right transformers are to shift the twist from the mass center to the contact point according to the known Euler formula: $(\mathbf{v}, \boldsymbol{\omega}) \mapsto (\mathbf{v} + [\boldsymbol{\omega}, \mathbf{r}], \boldsymbol{\omega})$, where the vector $\mathbf{r}$ begins at the corresponding center of mass and ends at the contact point. Reciprocally the wrenches shift to the body mass center from point of the contact in a following way: $(\mathbf{F}, \mathbf{M}) \mapsto (\mathbf{F}, \mathbf{M} + [\mathbf{r}, \mathbf{F}])$. As one can see easily the transformers conserve the power.

Central transformer is responsible for the transfer to orthonormal base at the contact point with the common normal unit vector and two others being tangent ones to both contacting bodies' surfaces supposed regular enough. For definity we interpret here the

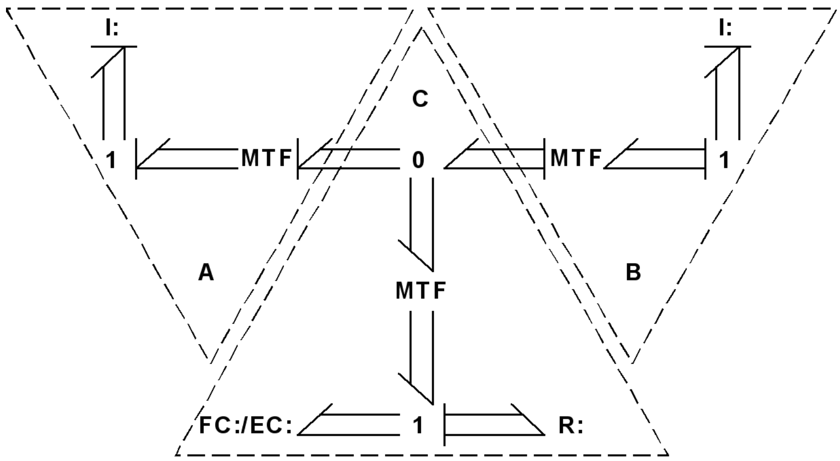

Figure 2: Architecture of Constraint: Bond Graph Representation

case of usual contact interconnection between the bodies by their outer/inner surfaces. If the inertial coordinates of these vectors compose columns of the orthogonal rotational matrix Q then shifting from bottom to top across the transformer in Figure 2 we will have for the flow signals: $(\mathbf{v}, \boldsymbol{\omega}) \mapsto (Q\mathbf{v}, Q\boldsymbol{\omega})$. Likewise when shifting in a reverse direction we have a transformation of the efforts: $(\mathbf{F}, \mathbf{M}) \mapsto (Q^{-1}\mathbf{F}, Q^{-1}\mathbf{M})$ also conserving the power. Organization of the 0-junction depicted in Figure 2 provides a possibility to compute exactly the relative velocities at the constraint contact point.

Note that it is a usual practice to attach the inertance element to 1-junction, in particular because of its causality nature, see for example [10, 11]. Figure 2 in some degree can remind us an element of the lumped model for the flexible beam dynamics.

Causality for some multibonds inside the constraint object is defined individually for each particular scalar bond [12] depending on the type of the constraint and is assigned finally after the whole MBS model compilation. For instance, if the constraint is of the slipping type at a contact then supposing decompositions of the relative velocities and contact forces $\mathbf{v} = \mathbf{v}_n + \mathbf{v}_\tau$, $\mathbf{F} = \mathbf{F}_n + \mathbf{F}_\tau$ we have the following flow constraint, element FC, $\mathbf{v}_n = \mathbf{0}$ representing one scalar kinematic equation for the normal relative velocity, and the effort constraint, element EC, $\mathbf{F}_\tau = \mathbf{0}$, $\mathbf{M} = \mathbf{0}$ representing two scalar equations for the tangent contact force plus three scalar equations for the contact torque. Nonzero tangent force at the contact may arise due to the resistive element, see the bottom right multibond. If we will continue to build the bond graph model for the whole MBS in a proposed way then finally we can arrive exactly to the so-called canonical junction structure [12] useful for the formal procedures of the bond graph optimal causality assignment. For this we have to add an intermediate 0-junctions for elements attached to 1-junction in the constraint component C, see Figure 2.

Leaving some multibonds without the causality assignment and trusting this work to compiler we apply a so-called acausal modeling [13, 14]. On the other hand if we will

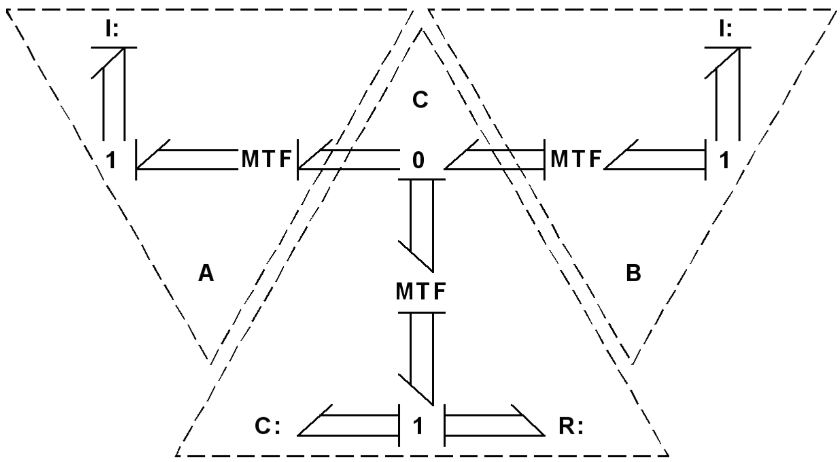

Figure 3: Bond Graph of Constraint with Compliance

act in a manner close to the real cases of constraints with the flexibility then instead of the constraint elements FC/EC, we have to use an element of the compliance with the causality uniquely determined, see Figure 3.

Further we analyze one example of the constraint frequently occurring in engineering applications: we consider an object classification of the joint constraint.

3 IMPLEMENTATION OF THE JOINT CONSTRAINT

For simplicity and clearness we will apply the component library to simulate the dynamics of MBSs with bilateral constraints [1]. Application of the components for the unilateral constraints [7] doesn't change anything in principle. The only difference is that dynamics of the moving bodies becomes more complicated. For example in the latter case a vehicle under simulation gets an ability to bounce over the uneven surface it rolls on. In addition, its wheels can slip while moving. Thus in frame of the current paper we suppose that nonholonomic constraints implemented exactly, without any slip or separation with respect to (w. r. t.) the surface.

Remind that according to our technology of the constraint construction [1] two connected bodies are identified by convention with the letters A and B fixed for each body. All kinematic and dynamic variables and parameters concerned one of the bodies are equipped with the corresponding letter as a subscript.

Class `Joint` plays a key role in the future model of a vehicle we will build. `Joint` is a model derived from the base class `Constraint`. Remind [7] that in order to make a complete definition of the constraint object behavior for the case of rigid bodies one has to compose a system of twelve algebraic equations w. r. t. to twelve coordinates of vectors $\mathbf{F}_A$, $\mathbf{M}_A$, $\mathbf{F}_B$, $\mathbf{M}_B$ constituting the wrenches acting upon the connected bodies.

First six equations always present in the base model `Constraint` due to Newton's third law. For definity suppose these six equations are used to express six components of $\mathbf{F}_B$,

$\mathbf{M}_B$ depending on $\mathbf{F}_A$, $\mathbf{M}_A$. Thus six components of $\mathbf{F}_A$, $\mathbf{M}_A$ remain as unknowns. To determine them each constraint of rigid bodies need in six additional independent algebraic equations. These equations can include components of force and torque directly, or be derived from the kinematic relations corresponding to specific type of the constraint.

In the case of the joint constraint being investigated here let us represent the motion of the body B as a compound one consisting of the body A transporting motion w. r. t. an inertial frame of reference which is similar to the Modelica Standard MultiBody Library model `World`, and a relative motion w. r. t. the body A. An absolute motion is one of the body B w. r. t. inertial system.

Define the joint constraint with help of the following parameters: (a) a unit vector $\mathbf{n}_A$ defining in the body A an axis of the joint; (b) a vector $\mathbf{r}_A$ fixed in the body A and defining a point which constantly stays on the axis of the joint; (c) a vector $\mathbf{r}_B$ fixed in the body B and defining a point which also constantly stays on the axis of the joint. The main task of the base joint class is to keep always in coincidence the geometric axes fixed in each of the bodies.

First of all one has to compute the radii vectors of the points fixed in the bodies w. r. t. inertial system

$$\mathbf{R}_\alpha = \mathbf{r}_{O_\alpha} + T_\alpha \mathbf{r}_\alpha \quad (\alpha = A, B),$$

where [7] $\mathbf{r}_{O_\alpha}$ is the position of the α-th body center of mass, T_α is its current matrix of rotation. The joint axis has the following components

$$\mathbf{n}_{Ai} = T_A \mathbf{n}_A$$

in the inertial frame of reference. According to the equation for relative velocity for the marked point of the body B defined by the position $\mathbf{R}_B$ we have

$$\begin{aligned} \mathbf{v}_{Ba} &= \mathbf{v}_{Be} + \mathbf{v}_{Br}, \\ \mathbf{v}_{Ba} &= \mathbf{v}_{O_B} + [\boldsymbol{\omega}_B, T_B \mathbf{r}_B], \\ \mathbf{v}_{Be} &= \mathbf{v}_{O_A} + [\boldsymbol{\omega}_A, \mathbf{R}_B - \mathbf{r}_{O_A}], \end{aligned} \tag{1}$$

where $\mathbf{v}_{Ba}$, $\mathbf{v}_{Be}$, $\mathbf{v}_{Br}$ are an absolute, transporting, and relative velocities of the body B marked point, $\boldsymbol{\omega}_A$, $\boldsymbol{\omega}_B$ are the bodies angular velocities.

Furthermore, according to the computational experience of the dynamical problems simulation the precompiler work is more regular if the kinematic equations are expressed directly through accelerations. Indeed, otherwise the compiler tries to perform the formal differentiation of equations for the velocities when reducing an index of the total DAE system. Frequently this leads to the problems either in time of translation or when running the model.

In the first case usually diagnostics of the compiler essentially helps the developer. In the second case the model has an unpredictable behavior, and only manual preliminary reduction “regularizes” the simulation process. Thus we differentiate equations (1) and obtain an equations for the relative linear acceleration in the form

$$\begin{array}{l}\mathbf{a}_{Ba} = \mathbf{a}_{O_B} + [\boldsymbol{\epsilon}_B, T_B\mathbf{r}_B] + [\boldsymbol{\omega}_B, [\boldsymbol{\omega}_B, T_B\mathbf{r}_B]]\,, \\ \mathbf{a}_{Be} = \mathbf{a}_{O_A} + [\boldsymbol{\epsilon}_A, \mathbf{R}_B - \mathbf{r}_{O_A}] + [\boldsymbol{\omega}_A, [\boldsymbol{\omega}_A, \mathbf{R}_B - \mathbf{r}_{O_A}]]\,, \\ \mathbf{a}_{Ba} = \mathbf{a}_{Be} + 2\,[\boldsymbol{\omega}_A, \mathbf{v}_{Br}] + \mathbf{a}_{Br}, \\ \mathbf{a}_{Br} = \mu\mathbf{n}_{Ai},\end{array} \tag{2}$$

where $\mathbf{a}_{Ba}$, $\mathbf{a}_{Be}$, $\mathbf{a}_{Br}$ are an absolute, transporting, and relative accelerations of the body B marked point, $\boldsymbol{\epsilon}_A$, $\boldsymbol{\epsilon}_B$ are the bodies angular accelerations.

We also need in an analytic representation of the conditions that the only projections of the bodies angular velocities and accelerations having a differences are ones onto the joint axis. Corresponding equations have a form

$$\begin{array}{rcl}\boldsymbol{\omega}_B & = & \boldsymbol{\omega}_A + \boldsymbol{\omega}_r, \\ \boldsymbol{\epsilon}_B & = & \boldsymbol{\epsilon}_A + [\boldsymbol{\omega}_A, \boldsymbol{\omega}_r] + \boldsymbol{\epsilon}_r, \\ \boldsymbol{\epsilon}_r & = & \lambda\mathbf{n}_{Ai},\end{array} \tag{3}$$

where $\boldsymbol{\omega}_r$, $\boldsymbol{\epsilon}_r$ are the relative angular velocities and accelerations.

Besides the kinematic scalars μ, λ we will need in their reciprocal values $F = (\mathbf{F}_A, \mathbf{n}_{Ai})$, $M = (\mathbf{M}_A, \mathbf{n}_{Ai})$ correspondingly. Note that the class described above is a partial one and can be used to produce any imaginable model of the joint type constraint. To obtain a complete description of the joint model one has to add to the behavioral section exactly two equations. One of them is to define one of the values μ, F (translatory case). Other equation is intended to compute one of the values λ, M (rotary case).

Regarding the general scheme depicted in Figure 2 we can conclude that the equations (1), (2), (3) together implement implicitly the constraint transformer to the joint local coordinate system and four scalar flow constraints forbidding relative translatory and rotary motions in the direction orthogonal to the joint axis. For derived classes only two free scalar bonds remain.

Here we encounter the known complementarity rules once more in a way similar to one described in [7]. In our context the variables in the pairs (μ, F), (λ, M) are mutually complement, where one of μ, λ is to be utilized for the flow constraint and one of F, M is used to compose the effort constraint. All the variables mentioned complete the set of constraints for the remaining yet unused joint axis creating thus two final scalar constraint elements in the bond graph of Figure 2.

Namely, the equations (2) implementing the Coriolis theorem for accelerations simultaneously implement, in an implicit manner, two scalar flow constraints, FC-elements, from the bottom left corner of the multi-bondgraph model in Figure 2. These flow constraints due to compiler restrictions constructed using accelerations instead of the velocities being used in a classic bond graph approach. The constraints have an obvious kinematic sense: they prevent the relative motion of the body B marked point in two directions normal to the joint axis fixed in the body A.

In addition, the equations (3) implement two other scalar flow constraints, this time for the rotary motion. These constraints forbid the relative rotation of the body B w. r. t. body A about two axes each normal to the joint axis mentioned above which is rigidly connected with the body A.

Note, that the construct of equations (2) and (3) is such that they allow the body B relative motion along and about the joint axis of the body A thus implementing the kinematic pair with two DOFs. Returning to Figure 2 of the general constraint multibond graph we can conclude that the vertical multibond attached to 0-junction implements flow variables corresponding to the relative body B motion w. r. t. body A in inertial coordinates. Such a description supposes an existence of the special coordinates reference frame connected with the body A at its joint constraint marked point. The transformation to these coordinates is implemented exactly via corresponding transformer, central in the triangle block C. The transformer itself nests in formulae of equations (2) and (3).

Consider several examples of the classes derived from the `Joint` model for the several particular types of joints. The model `FixedIdealJoint` is defined by the equations

$$\mu = 0, \quad M = 0$$

and prevents the relative motion along the joint axis but allows free rotation about it. It is exactly a revolute joint without any control for the rotary motion. The model `FreeIdealJoint` is defined by the equations

$$F = 0, \quad M = 0$$

permitting free translation along and free rotation about the joint axis.

Class `SpringIdealJoint` described by the equations

$$F = c\nu + d\dot{\nu}, \quad M = 0, \quad \ddot{\nu} = \mu$$

with an initial data $\nu(t_0) = 0$, $\dot{\nu}(t_0) = 0$ for the relative translatory position ν provides a viscoelastic compliance with the stiffness c and damping d. The rotary motion remains free. This model is useful to simulate almost rigid constraints to avoid the potential problems with so-called statically undefinable systems of forces acting upon the ideal rigid bodies.

The model `FixedControlledJoint` with the behavior defined by the equations

$$\mu = 0, \quad M = f(t, \varphi, \dot{\varphi}), \quad \ddot{\varphi} = \lambda \tag{4}$$

provides the rotating torque as a control effort with the prescribed control function $f(t, \varphi, \dot{\varphi})$. Initial data $\varphi(t_0) = \varphi_0$, $\dot{\varphi}(t_0) = \dot{\varphi}_0$ are prepared according to the initial data concerning the joint. From the bond graph viewpoint the second equation in (4) can be implemented as a combination of the source effort, compliance, and resistance elements. This type of joint corresponds to the `Revolute` joint constraint of Modelica

Standard Library from the `ModelicaAdditions` package. Such a joint can be driven by the electromotor.

The model `FreeSlideJoint` defined by the equations

$$F = 0, \quad \lambda = 0$$

provides free, without any resistance, relative sliding along the joint axis without any rotation about it. As one can see this is a prismatic type of joint.

We can reformulate the `FixedControlledJoint` model creating the model `FixedServoJoint` in a following useful way

$$\mu = 0, \quad \lambda = f\left(t, \varphi, \dot{\varphi}\right), \quad \ddot{\varphi} = \lambda$$

thus composing a kinematic restricting constraint, so-called servoconstraint. The function $f\left(t, \varphi, \dot{\varphi}\right)$ supposed as a prescribed one. Initial data for the angle φ of the relative rotation are prepared in the same way as for (4). It is clear one can create a lot of other different combinations of equations to construct the joint constraints needed in engineering applications.

The derived joint classes described here are to close the system of kinematic equations (2) and (3) completing them mainly by two scalar additional equations, each playing a role of an either FC-element, like $\mu = 0$, or EC-element, like $F = 0$. Any time to be able to construct a consistent system of equations for the total model we have to follow the guidelines of the complementarity rules.

These latter correspond to the notions of the bond graph theory in a natural way. Indeed, the theory of bond graphs is based on the energy interactions. Every our multibond being an energy/power conductor reflects complementarity by its twist/wrench duality. To close the total DAE system for the model under development we have to "close" or rather to "seal" each free scalar bond in EC/FC-element of the block C in Figure 2 by the corresponding one scalar equation for flow or effort variable. Thus here we outline the main rule to compose equations for the models of constraints for MBS of any type in a consistent way when applying the object-oriented approach.

See also papers [16, 17, 18] for particular cases of non-linear contact problems.

4 CONCLUSIONS

The brief list of concluding remarks is of the following one:

- A unified multibond graph representation of the MBS dynamics in a sufficiently simple way with the canonical junction structure is possible.

- The representation depicted in Figure 2 can be used as a guideline to construct the consistent system of DAEs in a systematic way. In other words we can say that multibond graph constructs like ones of Figure 2 are to be used as a regular basis for more informal object-oriented approach.

- An acausal modeling accelerates the model development releasing a project team from the problem of causality assignment if one takes into account some requirements like complementarity rules.
- Introducing the compliance into the model may be useful and efficient preserving the principal properties of the MBS like anholonomity etc.

5 ACKNOWLEDGEMENT

This work has been prepared at MAI and the Lomonosov MSU.

REFERENCES

[1] Kosenko, I. I., Stepanov, S. Y. and Gerasimov, K. V. Contact tracking algorithms in case of the omni–directional wheel rolling on the horizontal surface. *Multibody System Dynamics* (2019) **45**:273–292.

[2] *Modelica — a unified object-oriented language for physical systems modeling. Tutorial.* Modelica Association, (2000).

[3] *Modelica — a unified object-oriented language for physical systems modeling. Language specification.* Modelica Association, (2004).

[4] Karnopp, D. C., Margolis, D. L., Rosenberg R. C., *System dynamics: modeling and simulation of mechatronic systems. — 4th ed.* John Wiley & Sons, Hoboken, New Jersey, (2006).

[5] Karnopp D. C., The energetic structure of multibody dynamic systems. *Journal of the Franklin Institute*, (1978) **306**:165–181.

[6] Wittenburg, J. *Dynamics of systems of rigid bodies.* B. G. Teubner, Stuttgart, (1977).

[7] Kossenko, I. I., Implementation of Unilateral Multibody Dynamics on Modelica. *Proceedings of the 4th International Modelica Conference, Hamburg University of Technology, Hamburg–Harburg, Germany, March 7–8.* (2005) :13–23.

[8] Stramigioli, S., Blankenstein, G., Duindam, V., Bruyninckx, H., Melchiorri C., Power Port Concepts in Robotics. The Geometrical–Physical Approach. *Tutorial at 2003 IEEE International Conference on Robotics and Automation.* — IEEE, (2003).

[9] Paynter, H. M., *Analysis and Design of Engineering Systems.* — The M. I. T. Press: Cambridge, Massachusetts, (1961).

[10] Cellier, F. E., *Continuous System Modeling.* — Springer–Verlag: New York, (1991).

[11] Mukherjee, A., Karmakar, R., *Modelling and Simulation of Engineering Systems through Bondgraphs.* — Alpha Science International Ltd.: (2000).

[12] Golo, G., *Interconnection Structures in Port-Based Modelling: Tools for Analysis and Simulation. PhD Thesis.* — University of Twente: Enschede, The Netherlands, (2002).

[13] *Dymola. Dynamic Modeling Laboratory. User Manual. Volume 1. Version 7.3.* — Ideon Science Park, SE-223 70, Lund, Dynasim AB, (2009).

[14] *Dymola. Dynamic Modeling Laboratory. User Manual. Volume 2. Version 7.3.* — Ideon Science Park, SE-223 70, Lund, Dynasim AB, (2009).

[15] Duindam, V., Blankenstein, G., Stramigioli S., Port-Based Modeling and Analysis of Snakeboard Locomotion. *Sixteenth International Symposium on Mathematical Theory of Networks and Systems.* Katholieke Universiteit Leuven, Belgium, July 5–9 (2004)

[16] Kireenkov, A. A., Modelling of the Force State of Contact of a Ball Rolling along the Boundaries of Two Rails. (2021) AIP Conference Proceedings, 2343, Paper No 120002, DOI: 10.1063/5.0047955, ISSN: 0094243X, ISBN: 9780735440883.

[17] Kireenkov, A. A., Preface: Mathematical Models and Investigations Methods of Strongly Nonlinear Systems. (2021) AIP Conference Proceedings, 2343, Paper No 120001, DOI: 10.1063/5.0047954, ISSN: 094243X, ISBN: 9780735440883.

[18] Kireenkov, A. A., Zhavoronok, S. I., Nushtaev, D. V. On Tire Models Accounting for both Deformed State and Coupled Dry Friction in a Contact Patch. (2021) Computer Research and Modeling, 13 (1), pp. 163-173. DOI: 10.20537/2076-7633-2021-13-1-163-173, ISSN: 20767633.

.