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GLOBAL SENSITIVITY EVOLUTION OF VERTICAL RIDE QUALITY FOR HIGH-SPEED MAGLEV TRAINS

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Abstract. This study investigates the influence of structural parameter randomness on the ride quality of high-speed maglev train levitation systems, aiming to provide a theoretical foundation for structural optimization and control strategy design. The complex levitation system is first simplified into a single electromagnet levitation system incorporating secondary suspension, and the Sperling index is employed to assess ride quality quantitatively. Based on the probability density evolution method (PDEM), the probability density function (PDF) of the Sperling index is estimated. Subsequently, global sensitivity analysis is conducted using the Fréchet derivative, and the results are compared with those obtained using the conventional Sobol's method. The findings show that PDEM achieves a mean absolute error of only 0.0419 compared to the Monte Carlo method (MCM) with an 806-fold improvement in computational efficiency. The sensitivity rankings obtained from the method based on Fréchet derivative are consistent with those from the Sobol's method and further reveal the directional impact of parameters on the Sperling index. Overall, the electromagnetic winding turn, the magnet area, and the quality of the levitation electromagnet are identified as the key structural parameters influencing ride quality.

1 Introduction

Maglev trains are considered a promising mode of advanced ground transportation due to their advantages of low noise, low emissions, minimal maintenance requirements, and zero derailment risk [1]. As operating speeds increase, the interaction forces between the train and the guideway intensify, placing greater demands on the ride quality and presenting new challenges for structural design and control optimization [2]. Therefore, conducting in-depth studies on the ride quality of high-speed maglev trains and analyzing the influence of structural parameters is of significant engineering relevance. However, the levitation system of a high-speed maglev train is inherently a highly complex stochastic nonlinear system [3]. Its structural parameters remain uncertain due to inherent manufacturing imperfections and inevitable wear or degradation occurring during operation, making it difficult to estimate performance using deterministic methods accurately [4, 5]. Moreover, the influence of input parameter randomness on ride quality is directional. Understanding the magnitude and direction of these influences is crucial for structural analysis and optimal design under uncertainty [6].

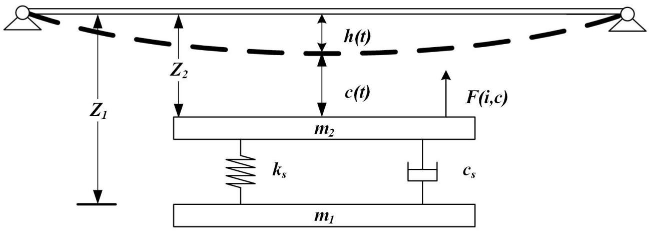


Figure 1: Structure of the single electromagnet levitation system.

Parameter sensitivity reflects how variations in input parameters affect the system output, and the method of identifying parameter importance by quantifying the contribution of parameter uncertainty to output variability is known as sensitivity analysis [7, 8]. Among existing methods, global sensitivity analysis methods such as the Sobol's method have been widely applied in train performance evaluations [9, 10, 11]. However, these methods only provide important identification with non-negative sensitivity indices. As a result, these methods do not provide information about the direction of influence, whether a parameter contributes positively or negatively to system performance [12].

A new approach combining the PDEM with Fréchet derivatives has emerged to overcome this limitation [13]. This method first estimates the PDF of the system's Sperling index using PDEM and then derives global sensitivity indices via Fréchet derivatives [14]. This enables the quantification of parameter importance and the identification of the direction in which each parameter influences system performance, thereby enhancing the usefulness of the analysis [15].

This study establishes a vertical dynamic model of a single electromagnet levitation system with secondary suspension to analyze the impact of structural parameter uncertainties on the Sperling index. The PDEM is employed to approximate the PDF of the Sperling index, and the results are compared with those from the MCM. Subsequently, global sensitivity analysis uses the Fréchet derivative to identify key structural parameters and determine their directional influence on ride quality. These results are further validated against those obtained from the Sobol's method. This paper is organized as follows: Section 2 introduces the mathematical model of the levitation system and the definition of the Sperling index. Section 3 presents the theoretical framework of the global sensitivity analysis method based on PDEM and the Fréchet derivative. Section 4 discusses and compares the results of the parameter sensitivity analysis.

2 Levitation system on the high-speed maglev train

2.1 Modeling of the levitation system

High-speed maglev trains consist of three to five carriages, with each carriage equipped with four levitation chassis. Each chassis contains eight levitation units, which support the entire train to enable high-speed operation [16]. Since this study focuses on the vertical ride quality of the levitation system, the complex system is simplified to a single electromagnet levitation system incorporating a secondary suspension, which is illustrated in Figure 1. In Figure 1, m_1 denotes the load quality of the secondary suspension and m_2 represents the quality of the levitation electromagnet. The levitation gap between the levitation electromagnet and the track is denoted by $c(t)$. The track displacement is represented by $h(t)$. The parameter Z_1 represents the displacement of the train body, while Z_2 denotes the displacement of the levitation chassis. c_s and k_s are the damping and stiffness of the secondary suspension, respectively. The positive reference direction in the system is downward.

According to Newton's second law, the vertical dynamic equation of the single electromagnet levitation system with secondary suspension can be formulated as follows

$$\begin{cases} m_1 \frac{\mathrm{d}^2 Z_1}{\mathrm{d}t^2} = m_1 g - F_0 - k_s\left(Z_1 - Z_2\right) - c_s\left(\frac{\mathrm{d}Z_1}{\mathrm{d}t} - \frac{\mathrm{d}Z_2}{\mathrm{d}t}\right), \\ m_2 \frac{\mathrm{d}^2 Z_2}{\mathrm{d}t^2} = -F(i,t) + m_2 g + F_0 + k_s\left(Z_1 - Z_2\right) + c_s\left(\frac{\mathrm{d}Z_1}{\mathrm{d}t} - \frac{\mathrm{d}Z_2}{\mathrm{d}t}\right), \\ F(i,t) = \frac{u_0 {N_1}^2 A_1}{4}\left[\frac{i(t)}{c(t)}\right]^2, \\ Z_2 = h(t) + c(t). \end{cases} \tag{1}$$

where F_0 is the force of the secondary suspension at the equilibrium point, $F(i,t)$ is the electromagnetic force, u_0 is the magnetic permeability of vacuum, N_1 is the number of electromagnetic winding turns, A_1 is the magnet area, $i(t)$ is the current, and $h(t)$ denotes the track irregularity.

2.2 Ride quality evaluation index of the levitation system

There is no dedicated dynamic performance evaluation standard for maglev train systems. As a result, the dynamic performance of maglev trains is assessed based on the evaluation criteria used for standard railway vehicles. The Sperling index provides a quantitative assessment of the running quality of railway vehicles [17]. Therefore, in this study, the Sperling index is adopted to evaluate the vertical ride quality of the high-speed maglev train levitation system. The calculation formula for the vertical Sperling index is given as follows

$$W = 0.896\left(\frac{a_m^3}{f}F(f)\right)^{1/10}, \tag{2}$$

where W is the Sperling index, a_m is the peak value of vibration acceleration, f is the vibration frequency, and $F(f)$ is the frequency weighting correction factor related to the vibration frequency. For vertical acceleration, the frequency weighting function $F(f)$ is given by

$$F(f) = \begin{cases} 0.325 f^2, & 0.5 \le f \le 5.9 \text{ Hz}, \\ \frac{400}{f^2}, & 5.9 < f \le 20 \text{ Hz}, \\ 1, & f > 20 \text{ Hz}. \end{cases} \tag{3}$$

After obtaining the Sperling index at different frequencies using Equations 2 and 3, the overall Sperling index is calculated using Equation 4 as

$$W_{\text{total}} = \left(W_1^{10} + W_2^{10} + W_3^{10} + \cdots + W_n^{10}\right)^{1/10}, \tag{4}$$

where W_{total} denotes the total Sperling index, and W_1 to W_n represent the Sperling index at different frequencies.

3 A global sensitivity analysis framework based on PDEM and the Fréchet derivative

3.1 Probability density evolution method

For the dynamic Equation 1 of the levitation system, let Θ denotes the random structural parameter of the train, and X denotes the Sperling index. Let $P_{X\Theta}(x,\theta,t)$ be the joint PDF of $(X(t),\Theta)$. According to the principle of probability conservation, it satisfies the following generalized probability density evolution equation [18]

$$\frac{\partial P_{X\Theta}(x,\theta,t)}{\partial t} + \dot{X}(\theta,t)\frac{\partial P_{X\Theta}(x,\theta,t)}{\partial x} = 0. \tag{5}$$

The boundary and initial conditions of the levitation system are given in Equation 6. Under these conditions, Equation 5 can be solved, and by further integration using Equation 7, the PDF of the Sperling index can be obtained.

$$\begin{cases} P_{X\Theta}(x,\theta,t_0) = \delta(x-x_0)P_\Theta(\theta), \\ P_{X\Theta}(x,\theta,t)|_{X\to\pm\infty} = 0. \end{cases} \tag{6}$$

$$P_X(x,t) = \int P_{X\Theta}(x,\theta,t)dx, \tag{7}$$

where $\delta(\cdot)$ is the Dirac function, $P_\Theta(\theta)$ denotes the joint PDF of the input random structural parameters of the train, and $P_X(x,t)$ is the PDF of the Sperling index.

3.2 Global sensitivity based on Fréchet derivative

The sensitivity analysis method based on the Fréchet derivative can separately obtain the effects of different distribution parameters of the train's random structural parameters on ride comfort. Let the joint PDF of the input random structural parameters be denoted by $P_{\boldsymbol{\Theta}}(\boldsymbol{\theta};\boldsymbol{\zeta})$, where $\boldsymbol{\zeta} = (\zeta_1,\zeta_2,\cdots,\zeta_s)^T$ is the vector of distribution parameters corresponding to the input random variables. According to previous studies [19], the random structural parameters in this study can be assumed to follow Gaussian distributions. Therefore, the distribution parameter vector consists of the means μ and standard deviations σ. Let n denote the number of random structural parameters, then

$$\boldsymbol{\zeta} = (\zeta_1,\zeta_2,\cdots,\zeta_s)^T = (\mu_1,\sigma_1,\mu_2,\sigma_2,\cdots,\mu_n,\sigma_n)^T.$$

Assuming that all distribution parameters are mutually independent, when a small perturbation $\Delta\zeta_j$ is applied to the j-th distribution parameter while keeping all others fixed. The parametric expression of the global sensitivity index based on the Fréchet derivative is given by Equation 8.

$$\mathcal{F}_{\psi,j} = \frac{\partial P_X(x;\zeta)/\partial\zeta_j}{\|\partial P_\Theta(\theta;\zeta)/\partial\zeta_j\|_V}, j=1,2,\cdots,s, s=2n. \tag{8}$$

$$\|\partial P_\Theta(\theta;\zeta)/\partial\zeta_j\|_V = \frac{1}{2}\int_{\Omega_\Theta} |\partial P_\Theta(\theta;\zeta)/\partial\zeta_j| d\theta. \tag{9}$$

The sign of the Fréchet derivative can be used to determine the direction of the distribution parameters that influence the Sperling index. Specifically, when a small positive perturbation is applied to an input distribution parameter, a positive Fréchet derivative indicates an increase in the probability density within the corresponding range of the Sperling index. A negative Fréchet derivative indicates a decrease in the probability density in that range. The opposite holds for a small negative perturbation of the distribution parameter. This allows us to characterize the trend of changes in the PDF of the Sperling index to variations in the distribution parameters.

The numerator in Equation 8 can be numerically approximated using the central difference method. Let the increment of $\boldsymbol{\zeta}$ along the ζ_j direction be $\Delta\zeta_j$ (typically taken as $\Delta\zeta_j = 10^{-2}\zeta_j$ or $10^{-3}\zeta_j$; in this study, $\Delta\zeta_j = 10^{-2}\zeta_j$). Equation 8 can be rewritten as

$$\mathcal{F}_{\psi,j} \approx \frac{P_X(x;\boldsymbol{\zeta}+\boldsymbol{e}_j\Delta\zeta_j) - P_X(x;\boldsymbol{\zeta}-\boldsymbol{e}_j\Delta\zeta_j)}{2\Delta\zeta_j\|\partial P_\Theta(\theta;\boldsymbol{\zeta})/\partial\zeta_j\|_V}, \tag{10}$$

where $\boldsymbol{e}_j$ is an s-dimensional unit vector, with the j-th element equal to 1 and all other elements equal to 0. $P_X(x;\zeta+\mathbf{e}_j\Delta\zeta)$ and $P_X(x;\zeta-\mathbf{e}_j\Delta\zeta)$ can be obtained by solving Equation 5 using the finite difference method. The global sensitivity index based on the Fréchet derivative can be obtained by solving Equation 10.

Applying the $\|\cdot\|_W$ norm to the global sensitivity index based on the Fréchet derivative, the importance measure S_j for the j-th input distribution parameter can be obtained. A larger value S_j implies that the corresponding input distribution parameter has a greater influence on the variability of the Sperling index [13].

$$\mathcal{S}_j = \|\mathcal{F}_{\psi,j}\|_W = \frac{\|P_X\left(x;\boldsymbol{\zeta}+\boldsymbol{e}_j\Delta\zeta_j\right) - P_X\left(x;\boldsymbol{\zeta}-\boldsymbol{e}_j\Delta\zeta_j\right)\|_W}{2\Delta\zeta_j\left\|\partial P_\Theta(\theta;\boldsymbol{\zeta})/\partial\zeta_j\right\|_V}. \tag{11}$$

$$\begin{aligned}&\|P_X\left(x;\boldsymbol{\zeta}+\boldsymbol{e}_j\Delta\zeta_j\right) - P_X\left(x;\boldsymbol{\zeta}-\boldsymbol{e}_j\Delta\zeta_j\right)\|_W\\ &=\frac{1}{2}\int_{\Omega_X}\left|P_X\left(x;\boldsymbol{\zeta}+\boldsymbol{e}_j\Delta\zeta_j\right) - P_X\left(x;\boldsymbol{\zeta}-\boldsymbol{e}_j\Delta\zeta_j\right)/\partial\zeta_j\right|\,dx.\end{aligned} \tag{12}$$

4 Results of the global sensitivity analysis

4.1 Numerical experiment configuration

The levitation system is an open-loop unstable system. This study employs a closed-loop feedback control strategy based on the current $i(t)$ in Equation (1) to achieve stable control of the levitation system [20]. In addition, the track irregularity in Equation (1) consists of both deterministic and stochastic components. The stochastic component is modeled as Gaussian white noise with a spectral density of $10^{-11}W/Hz$, presenting random track disturbances. A sinusoidal function can approximate the deterministic irregularity, as expressed in Equation 13[21].

$$y = A_{\mathrm{m}}\sin\left(2\pi\frac{vt}{L_m}\right), \tag{13}$$

where A_m represents the deterministic track irregularity amplitude, approximated as 0.001m. Similarly, L_m denotes the irregularity wavelength, estimated as 25m. Finally, v is the train speed and t is the operation time.

There are six structural parameters in the vertical dynamic Equation 1. These include the magnet area, the load quality of the secondary suspension, the quality of the levitation electromagnet, the secondary suspension stiffness, the secondary suspension damping, and the electromagnetic winding turns. This study aims to determine both the importance and direction of the influence of train random structural parameters on ride comfort. Since the sensitivity analysis method based on the Fréchet derivative can separately obtain the effects of different distribution parameters of the train's random structural parameters on ride comfort and the calculation procedures are identical. This paper focuses only on the influence of the mean values of the train's random structural parameters as an example for analysis. Before evaluating the ride quality, it is necessary to obtain the acceleration response. Figure 2 shows the vertical acceleration responses of the levitation system under 100 sets of randomly sampled parameters. Based on previous studies [19], the parameters are assumed to follow a Gaussian distribution with a coefficient of variation of 0.01, and the mean values of the parameters are listed in Table 1.

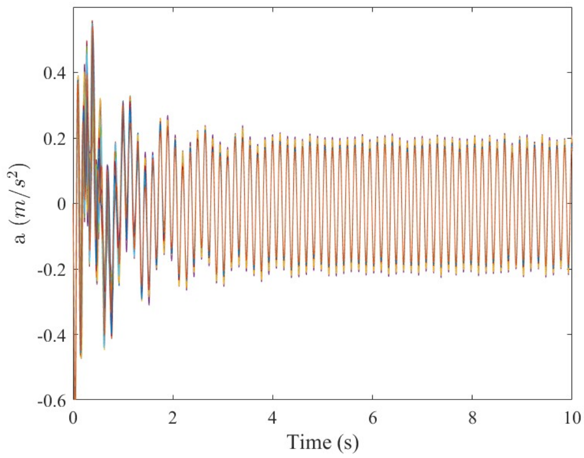


Figure 2: Vertical acceleration responses of levitation system with random structural parameters.

Table 1: Specifications of the electromagnetic levitation system model parameters.

Parameter	**Quantity**	**Unit**
Magnet area (A_1)	0.115	m^2
Load quality of the secondary suspension (m_1)	2000	Kg
Quality of levitation electromagnet (m_2)	750	Kg
Secondary suspension stiffness (k_s)	150000	N/m
Secondary suspension damping (c_s)	3000	N/(m/s)
Electromagnetic winding turns (N_1)	270	turn

4.2 Method validation

4.2.1 Probability density function

To validate the effectiveness and accuracy of the PDEM in evaluating the ride quality of high-speed maglev trains, the classical MCM is adopted as a reference. The MCM involves large-scale random sampling of uncertain system parameters, followed by computation of the system's acceleration response using the dynamic model of the levitation system. Subsequently, the Sperling index is calculated, and its PDF is estimated, serving as the benchmark reference for the validation of the PDEM.

In this study, a total of 10^5 Monte Carlo samples are generated, and the PDF of the Sperling index is obtained using the kernel density estimation method. Compared with MCM, the PDF results obtained by PDEM exhibit high consistency, with the mean absolute error is 0.0419, as shown in Figure 3. This confirms the accuracy of the PDEM in probabilistic ride quality analysis.

Moreover, regarding computational efficiency, the total simulation time required by MCM is approximately 258,754 seconds, whereas the PDEM only takes 321 seconds (approximately $1/806$ of MCM's runtime), demonstrating a significant improvement in computational efficiency. Therefore, the PDEM achieves a favorable balance between accuracy and efficiency, making it highly suitable for rapid probabilistic assessment of ride quality in high-speed maglev trains under multiparameter uncertainty.

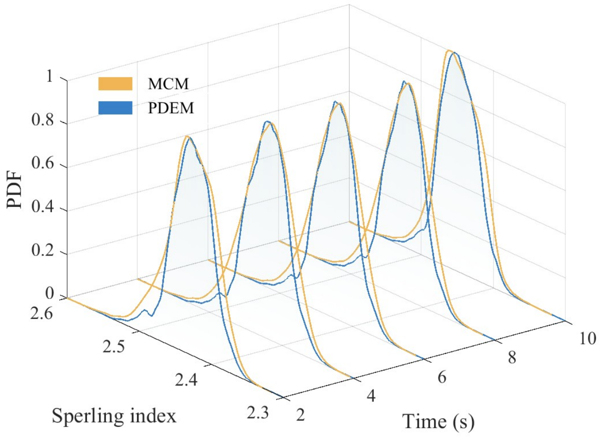
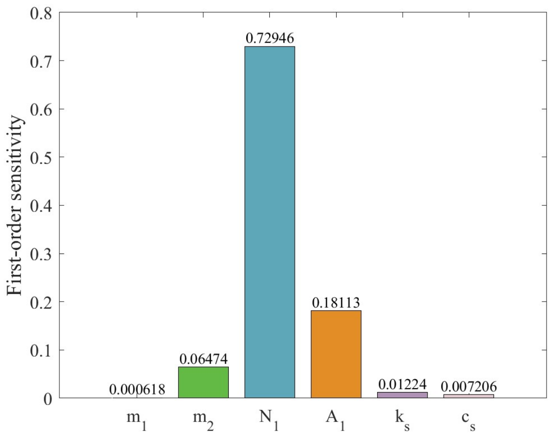


Figure 3: Comparison of instantaneous PDF distributions of the Sperling index

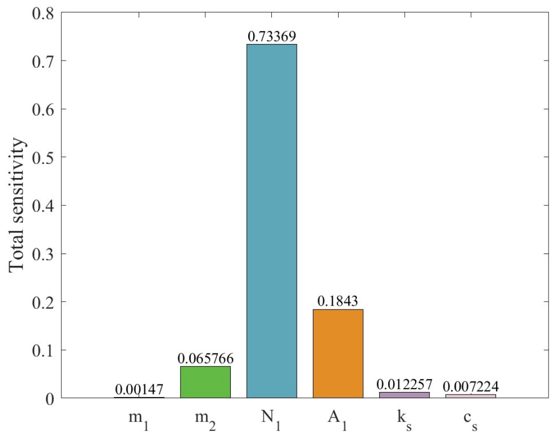
4.2.2 Importance identification

To further validate the reliability of the PDEM combined with Fréchet derivatives for sensitivity analysis of the levitation system in high-speed maglev trains, this study employs the classical Sobol's global sensitivity analysis method to quantitatively assess the influence of structural parameter uncertainties on the Sperling index.

At $t = 5$ s with a train speed of 600 km/h, the Sobol's sensitivity indices are shown in Figure 4. The results reveal that the influence ranking of the parameters on the Sperling index is as follows: $N_1 > A_1 > m_2 > k_s > c_s > m_1$. Since only N_1, A_1, and m_2 exhibit significant sensitivity indices (greater than 0.05), subsequent analysis focuses on these three dominant parameters.



(a) First-Order sensitivity.



(b) Totol sensitivity.

Figure 4: Global sensitivity analysis based on Sobol' method.

Based on the procedure outlined in Section 3.2, the global sensitivity measures of the Sperling index, obtained using the PDEM and the Fréchet derivative are calculated and summarized in Table 2. The results demonstrate a high degree of consistency between the importance rankings obtained from

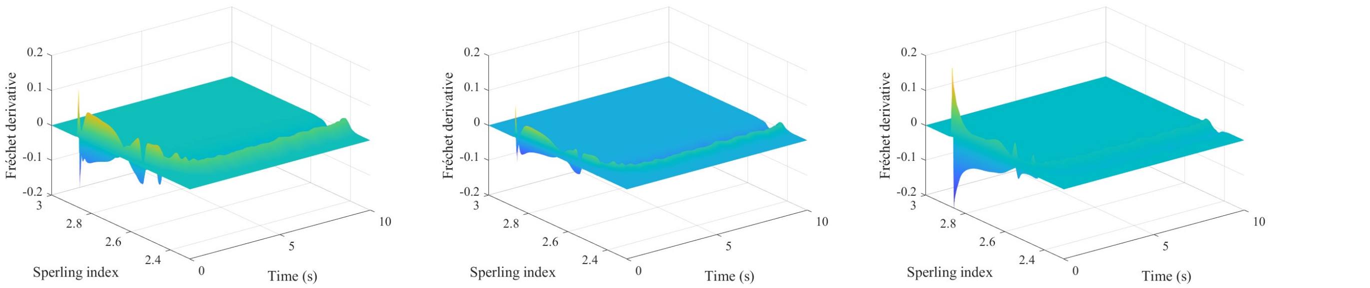
Table 2: Global sensitivity of structural parameters on the Sperling index.

Parameter	**Sobol' method (First-order sensitivity)**	**Fréchet derivative (Importance measure**$(\times 10^{-2})$**)**
N_1	0.73	0.16
A_1	0.18	0.07
m_2	0.06	0.04

the Fréchet derivatives and those derived using the Sobol' method, thereby validating that the PDEM combined with Fréchet derivatives can reliably characterize the sensitivity of system ride quality to uncertain parameters.

4.3 Direction sensitivity analysis of parameters based on Fréchet derivatives

Figure 5 shows the evolution of the Fréchet derivatives of the structural parameters to the Sperling index. For better clarity, the instantaneous Fréchet derivatives are presented in Figure 6. As shown in Figure6(a), the Fréchet derivative of N_1 initially exhibits positive values along the horizontal scale and then turns negative. This indicates that an increase in N_1 causes the probability density curve of the Sperling index to shift to the left. Therefore, the influence direction of N_1 on the Sperling index is negative, meaning that as N_1 increases, the Sperling index decreases, leading to improved ride quality. The analysis process for the remaining parameters is similar. By comprehensively considering both the importance and direction of the impact of each parameter on the Sperling index, the structural parameters can be reasonably optimized, under economic constraints, further to enhance the ride quality of the high-speed maglev train.



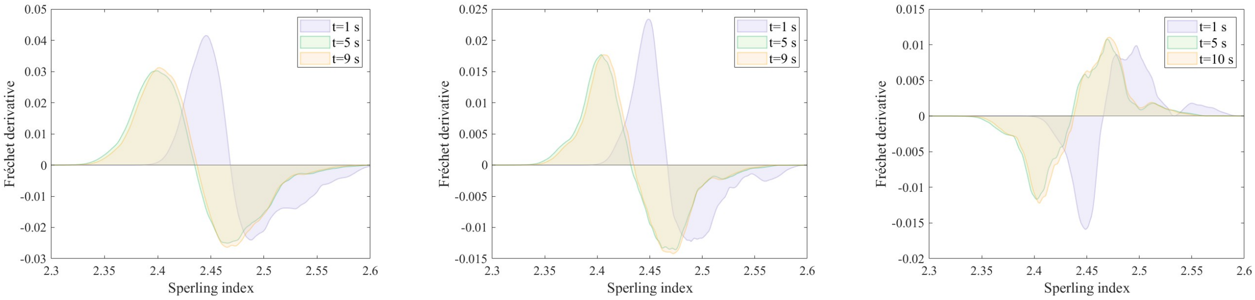
(a) Fréchet derivatives of N_1. (b) Fréchet derivatives of A_1. (c) Fréchet derivatives of m_2.

Figure 5: Evolution of Fréchet derivatives for Sperling index.

5 Conclusion

Based on the PDEM, this study conducted a global sensitivity analysis of the effects of random train structural parameters on the ride quality of the high-speed maglev train levitation system using Fréchet derivatives. The parameters significantly affecting ride quality were identified, and their influence directions were clarified. The main conclusions are as follows

(1) The global sensitivity analysis method based on PDEM and the Fréchet derivative improves the computational efficiency of PDF estimation by 806 times compared to the MCM, while



(a) Fréchet derivatives of N_1. (b) Fréchet derivatives of A_1. (c) Fréchet derivatives of m_2.

Figure 6: Instantaneous Fréchet derivatives for Sperling index.

maintaining high accuracy. Moreover, it enables the identification of the influence directions of the parameters on the Sperling index, which cannot be achieved by using the Sobol's method.

(2) Among the train structural parameters, the electromagnetic winding turn, the magnet area, and the quality of levitation electromagnet significantly impact ride quality. In contrast, the secondary suspension damping and stiffness, as well as the load quality of the secondary suspension, have minor effects.

(3) An increase in the electromagnetic winding turn or the magnet area leads to a decrease in the Sperling index, thus improving the train's ride quality. Conversely, an increase in the quality of levitation electromagnet causes the Sperling index to increase, thereby deteriorating ride quality.

(4) The global sensitivity analysis method based on PDEM and the Fréchet derivative can not only quantify the importance of input parameters on output responses but also reveal their influence directions, providing direct theoretical guidance for structural improvement and levitation control optimization of high-speed maglev trains.

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