

# Ductile-Fragile Transition: A Novel Comparison Between Fracture Mechanics of Materials and Framed Structures

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## Abstract

This paper deals with the issue of structural collapse considering an analogy between ductile-brittle transition of materials, taking into account the current literature which also considers the number of fragility and the stress intensification factor in the presence of crack, with extension on a larger scale involving framed structures subjected to increasing vertical loads.

It is evaluated the ductile-fragile transition in relation to concrete frames with different structural hierarchy (2x2, 5x5, 11x11).

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## 1 Introduction: Structural Collapse And Fracture Mechanics

In modern civil engineering, the concept of structural resistance is fundamental for ensuring the safety and functionality of buildings and infrastructures. Structural resistance is typically defined by two key design limits: the serviceability limit state (SLS), which ensures that a structure performs adequately under normal conditions without excessive deformations or vibrations, and the ultimate limit state (ULS), which determines the maximum load a structure can sustain before failure. While traditional engineering approaches emphasize these limits, real-world structural failures often reveal the need for a deeper understanding of how materials and structures behave under extreme conditions, particularly in the presence of defects, cracks, and progressive damage. To accurately predict and mitigate structural collapse, it is essential to move beyond conventional strength-based design and incorporate fracture mechanics principles, which focus on how cracks initiate and propagate within materials and structural systems. Fracture mechanics provides a framework for understanding why seemingly intact structures can suddenly fail due to stress concentration, material brittleness, or unforeseen loading scenarios.

The study of fracture mechanics traces back to the pioneering work of A.A. Griffith, who, in his landmark 1920 paper, introduced the concept of energy-based fracture criteria. Initially focusing on brittle materials such as glass, Griffith demonstrated that the presence of microscopic flaws significantly affects a material's strength. His work was instrumental in explaining why real materials often fail at stress levels much lower than their theoretical strength suggests. Although Griffith's research was primarily concerned with small-scale material failures, its implications extended far beyond laboratory specimens. Engineers soon recognized that the same principles applied to large-scale structures, including ships, bridges, and high-rise buildings. During the early 20th century, catastrophic failures such as brittle fractures in steel bridges and ship hulls highlighted the limitations of traditional strength-based design approaches, reinforcing the need for fracture mechanics as a critical engineering tool. Structural failure is rarely a sudden, isolated event; instead, it is often the result of progressive damage accumulation, where small defects grow and interact over time. This underscores the necessity of analyzing individual structural elements before scaling up to full-system behavior.

A comprehensive failure analysis must consider material behavior, geometric effects, load conditions, and hierarchical organization. Material behavior refers to the intrinsic properties of materials, such as ductility, brittleness, toughness, and fracture energy, which play a fundamental role in determining failure modes. Geometric effects concern the shape, dimensions, and stress distribution within a structural element, influencing where and how cracks form and propagate. Load conditions include external factors such as cyclic loading, dynamic impacts, temperature variations, and accidental overloads, which can accelerate structural deterioration. Finally, hierarchical organization describes the arrangement of structural components whether in a simple beam or a complex framed structure which affects how forces redistribute after initial damage. One of the most important aspects of modern structural analysis is stress concentration, which occurs when abrupt changes in geometry, such as notches, holes, or cracks, cause local increases in stress. These regions become weak points where failure is likely to initiate. Griffith's work demonstrated that as soon as a material exhibits a geometric discontinuity, the localized stress can exceed its theoretical strength, leading to crack initiation and eventual structural collapse.

Applying fracture mechanics principles to framed structures allows engineers to predict and control failure mechanisms more effectively. In particular, understanding the ductile-to-brittle transition in both materials and large-scale structures is crucial for designing safer buildings. The study of framed structures under increasing loads provides insight into how local failures such as the failure of a single beam or column can lead to progressive collapse if proper design strategies are not implemented. By drawing analogies between material-scale fracture mechanics and structure-scale collapse mechanisms, this research seeks to refine our understanding of robustness in civil engineering. The goal is to develop predictive models that not only assess the risk of brittle failure but also guide the design of more resilient structural systems, ensuring that they can withstand unexpected damage while maintaining overall stability. This study thus focuses on evaluating the ductile-brittle transition in reinforced concrete frame structures of varying complexity, analyzing how structural hierarchy influences failure resistance. By extending classical fracture mechanics equations to full-scale building systems, the research aims to establish a direct correlation between material-level failure criteria and structural-scale collapse behavior, providing a more comprehensive framework for structural safety and durability.

## **2 Ductile-Fragile Transition Of Materials**

The concept of ductile-to-brittle transition is a fundamental aspect of fracture mechanics and structural engineering, as it describes the shift in failure behavior from a gradual, energy-dissipating mode to a sudden, catastrophic rupture. This transition is influenced by multiple factors, including material properties, geometric characteristics, loading conditions, and environmental factors. In structural applications, understanding the ductile-to-brittle transition is essential for ensuring the safety and robustness of buildings and infrastructures, particularly in cases where extreme loading conditions or unforeseen damage could lead to progressive collapse.

The phenomenon of ductile-to-brittle transition is deeply rooted in the interplay between stress and deformation, as well as the presence and propagation of cracks within a material. Materials used in structural applications can generally be classified into two main categories: ductile and brittle.

Ductile materials, such as structural steel and certain metal alloys, exhibit a significant capacity for plastic deformation before failure. When subjected to increasing loads, these materials initially respond in a linear elastic manner, where stress is directly proportional to strain. As the load continues to increase, ductile materials enter a plastic deformation phase, in which they can undergo significant elongation, bending, or redistribution of internal stresses before reaching ultimate failure. This ability

to deform plastically allows ductile materials to absorb substantial amounts of energy, making them preferable in many engineering applications where resistance to sudden failure is critical.

By contrast, brittle materials, such as concrete, glass, and some high-strength ceramics, exhibit a limited ability to deform plastically before fracturing. Under increasing loads, these materials follow an initial linear elastic behavior similar to ductile materials, but instead of transitioning into a plastic deformation phase, they reach a critical stress threshold at which crack propagation leads to an abrupt loss of structural integrity. The presence of pre-existing flaws or defects within a material plays a crucial role in the ductile-to-brittle transition. In an idealized, defect-free material, failure would theoretically occur only when the atomic bonds reach their maximum stress capacity. However, real-world materials are far from perfect, containing microcracks, voids, inclusions, or other imperfections that serve as stress concentrators.

When a material is subjected to an external load, these imperfections cause localized stress intensification, significantly reducing the actual load-bearing capacity of the material. In ductile materials, plastic deformation mechanisms, such as dislocation movement, can help to redistribute these stresses and prevent catastrophic crack propagation. However, in brittle materials, stress concentration at the crack tip can exceed the material's critical stress intensity factor, leading to rapid crack growth and sudden failure.

Temperature is another key factor influencing the ductile-to-brittle transition, particularly in materials such as certain steels, where the transition occurs over a well-defined temperature range. At higher temperatures, atomic vibrations and dislocation movements facilitate plastic deformation, allowing the material to behave in a ductile manner. As temperature decreases, dislocation mobility is reduced, and the material becomes more prone to brittle fracture. This temperature-dependent behavior is commonly observed in ferritic steels, which can undergo a sharp transition from ductile to brittle failure at a specific transition temperature. Engineers account for this effect by conducting Charpy impact tests, which measure the energy absorbed by a material during fracture at various temperatures. Understanding this transition temperature is particularly important for structures exposed to cold environments, such as bridges, offshore platforms, and pipelines, where brittle failure poses a significant risk. The size and geometry of a structural element also influence its failure mode and the transition between ductile and brittle behavior. Larger structural components tend to exhibit a greater propensity for brittle failure due to the statistical probability of containing critical flaws. In fracture mechanics, this is described by the concept of size effect, where the nominal strength of a structure decreases as its dimensions increase.

When comparing small-scale laboratory specimens to full-scale structural components, it is often observed that larger specimens are more susceptible to brittle fracture because the probability of encountering a critical flaw increases with volume. Similarly, the shape of a structural element affects stress distribution and the likelihood of crack initiation. Sharp corners, notches, and sudden changes in cross-section create regions of high stress concentration, where cracks are more likely to form and propagate. Engineers mitigate these effects by incorporating design features such as fillets, rounded edges, and reinforcement details that help to reduce stress concentration and enhance the overall toughness of the structure. Another important aspect of the ductile-to-brittle transition is the role of fracture energy, which represents the total energy required to propagate a crack through a material. This energy is directly related to the material's ability to absorb and dissipate stress before failure. In ductile materials, fracture energy is relatively high, as significant plastic deformation occurs before crack propagation. In brittle materials, fracture energy is much lower, meaning that cracks can propagate with minimal energy input. Experimental studies have shown that the fracture energy of a material decreases as the specimen size increases, further reinforcing the notion that larger structures are more prone to brittle failure.

The presence of reinforcement, such as steel bars in concrete structures, can improve fracture energy by providing additional mechanisms for energy dissipation, such as crack bridging and stress redistribution. In structural applications, the ductile-to-brittle transition is not only a material-level phenomenon but also extends to entire structural systems. Reinforced concrete frame structures, for example, exhibit varying degrees of ductility depending on their design, material composition, and load distribution. In multi-story buildings, ductility is a key consideration in seismic design, where structures must be capable of dissipating seismic energy through controlled plastic deformations. Engineers achieve this by designing structural elements with appropriate reinforcement detailing, ensuring that plastic hinges form in predetermined locations, allowing the structure to absorb energy without experiencing sudden collapse. However, if a structure lacks sufficient redundancy or has an unfavorable load path, it may experience a brittle failure mechanism, leading to disproportionate collapse. Experimental and numerical studies on frame structures have shown that structural hierarchy plays a significant role in determining the transition between ductile and brittle failure modes. In a hierarchical frame structure, where larger primary elements support smaller secondary elements, stress redistribution mechanisms enhance overall robustness. By contrast, in a more heterogeneous frame structure, where elements of varying stiffness and strength are interconnected, stress localization can lead to premature brittle failure.

The concept of fragility number, introduced in analogy with fracture mechanics, provides a quantitative measure of a structure's susceptibility to brittle collapse. A lower fragility number indicates a more ductile response, while a higher fragility number suggests a greater likelihood of sudden failure. The transition from ductile to brittle collapse in framed structures is influenced by damage extent and hierarchy. When a structure experiences localized damage, such as the sudden removal of a column or beam, the redistribution of internal forces determines whether the structure fails gradually or abruptly. In highly redundant and hierarchical structures, the load can be effectively redistributed to undamaged elements, preventing immediate collapse. However, in less hierarchical structures, the loss of a key load-bearing element can trigger a chain reaction, leading to progressive collapse. This behavior is analogous to crack propagation in brittle materials, where stress concentration at the crack tip accelerates failure.

The findings from fracture mechanics studies can be extended to structural analysis to improve design strategies and enhance resilience against brittle failure. Engineers incorporate ductility-enhancing measures, such as confinement reinforcement in concrete, energy dissipation devices in seismic design, and strategic placement of structural fuses that localize damage while preserving overall stability. By understanding the fundamental principles of ductile-to-brittle transition at both the material and structural levels, it becomes possible to develop safer, more reliable engineering solutions for modern infrastructure.

The transition from ductile to brittle collapse in materials, considering for example a beam subject to central concentrated load  $P$ , depends on a dimensional effect and in particular on the length of the crack  $a_0$ . For crack lengths  $a < a_0$  the collapse propagation occurs in a ductile manner, while for  $a > a_0$  the brittle collapse precedes the plastic collapse. The characteristic length can be expressed through the following relationship which takes into account the stress intensification factor in the vicinity of the crack:

$$a_0 = \frac{1}{\pi} \frac{K_I^2}{\sigma_P^2} \quad (1)$$

where  $K_I$  is the stress intensification factor, while  $\sigma_P$  is the ultimate resistance of the material (in this case tensile strength).

In particular, with reference to [2] it is possible to deduce the following relationship which links the various geometric characteristics of the beam, with crack width  $a$ , ultimate plastic load  $P_{max}$  and tensile strength  $\sigma_P$ :

$$\frac{P_{max}L}{\sigma_P t H^2} = \left(1 - \frac{a}{H}\right)^2 \quad (2)$$

where  $L$  is the length of the beam,  $H$  is the height and  $t$  is the thickness.

### 3 Analogies With Progressive Collapse Of 2D Frames

The study of progressive collapse in 2D framed structures provides a crucial opportunity to explore the analogies between material fracture mechanics and structural failure mechanisms on a larger scale. By examining reinforced concrete frames of varying structural hierarchies, this research aims to establish how the principles governing crack propagation in brittle materials can be extended to the behavior of full-scale buildings subjected to increasing loads and localized damage. The primary objective is to analyze how different levels of structural hierarchy influence the transition from ductile to brittle failure and to quantify the structural fragility associated with various configurations. The study focuses on three specific frame arrangements 2x2, 5x5, and 11x11 each representing different levels of hierarchy, ranging from highly organized and redundant to more heterogeneous and fragmented. Through numerical simulations and comparative analysis, this research highlights the critical role of hierarchy in determining a structure's ability to redistribute loads after localized damage and resist progressive collapse. To ensure a comprehensive assessment, all frames are designed with the same overall dimensions but differ in the number and arrangement of their constituent elements. The 2x2 frame consists of fewer but larger primary structural elements, creating a robust and highly hierarchical system. The 5x5 and 11x11 configurations, on the other hand, introduce a greater number of smaller elements, leading to a more heterogeneous distribution of structural mass and stiffness. These differences in organization play a fundamental role in determining how each structure responds to external loads, particularly when subjected to sudden damage events. The primary focus of the analysis is on evaluating the ultimate plastic load in both intact and damaged conditions, as well as examining how the width of the damage zone affects the ability of each frame to withstand further loading before collapse.

2D framed structures are examined in Figure 1, which are composed of reinforced concrete with a high plastic strain and rotation capacity, as an application of the suggested methodology. The structures are composed of varying numbers of structural cells ( $n^2$ ), despite having the same overall size. The frames can be understood as a hierarchical reorganization of the frame with  $n = 11$  when  $n = 2$  and  $n = 5$  are used. This is achieved by using a primary structure composed of fewer but larger structural elements. In order to achieve this, we proportionately set the cross sectional area and reinforcement of the beams and columns to their respective lengths,  $L$  and  $H$ . In this manner, for varying  $n$ , the structural elements' slenderness remains constant. Now frame's hierarchical level is referred as  $1/n$ . Details on the parameters of the materials, cross sections, and design strategy can be found in [6, 7]. The frames are subjected to the sudden removal of beams and columns within a damage area, dotted in Figure 1. Details of damages scenarios can be found in [3].

Within the scope of this this paper it is treated the general case of the CB damage scheme.

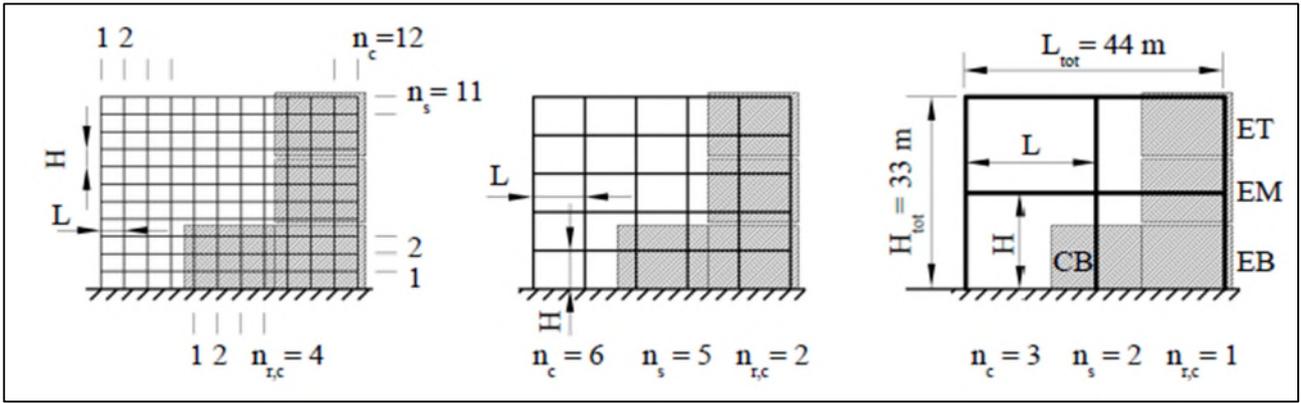


Figure 1 Studied frames and damage positions (dashed area). B = bottom, T = top, C = central, E = external.

It is considered  $f_c = \sigma_P$  35MPa: very strong columns inducing a collapse with bending mode mechanism [3].

Returning to the analogy with materials treated in equation (2) of Paragraph 2, this formulation can also be extended for the introduced frame structures.

In particular:

	2x2	5x5	11x11
$P_{max}$ [kN]	433125	173250	78750
L [m]	22	8,8	4
$\sigma_p$ [kN/m <sup>2</sup> ]	35000	35000	35000
t [m]	1	1	1
H [m]	16,5	6,6	3
a [m]	0	0	0

Table 1 Parameters of intact structures

	2x2	5x5	11x11
$P_{max}$ [kN]	54141	15593	3986
L[m]	22	8,8	4
$\sigma_p$ [kN/m <sup>2</sup> ]	35000	35000	35000
t [m]	1	1	1
H [m]	16,5	6,6	3
a [m]	8,3	2,6	1,1

Table 2 Parameters of damaged structures

With reference to the parameters inserted in Table 1 and Table 2 it is thus possible to diagram in Figure 2 the variation of the dimensionless ultimate plastic load  $P_{max}$  with respect to the width of the damage zone  $a$ , as the hierarchy of the structures considered varies. With the same size of the damaged area, the ultimate plastic load  $P_{max}$  is greater in the hierarchical structure, 2x2, rather than in the more heterogeneous and less hierarchical ones, 5x5, 11x11.

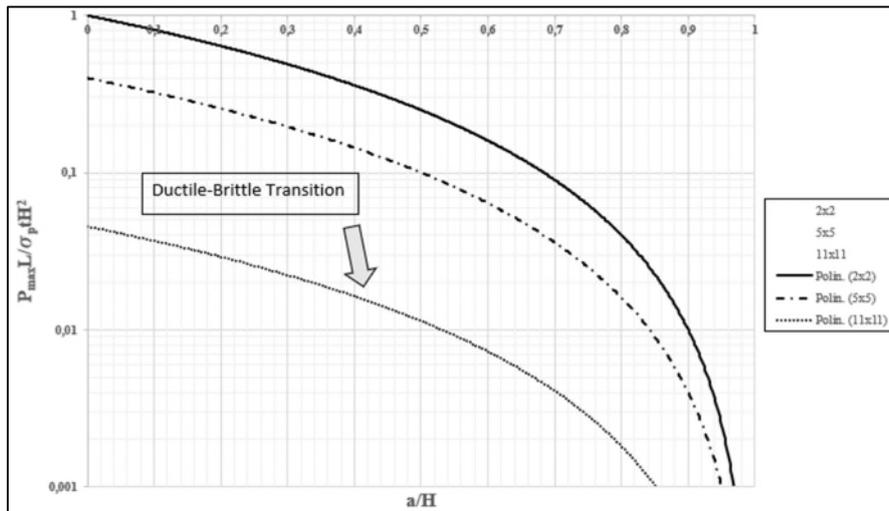


Figure 2 Transition from Ductile to Brittle behaviour in studied frames with damage CB

The first major finding of the study is that structural hierarchy significantly influences the magnitude of collapse. In the intact state, the 2x2 frame exhibits the highest ultimate load capacity, followed by the 5x5 and 11x11 frames, which display progressively lower values. This result is consistent with the principles of hierarchical organization in structural engineering, where larger primary elements enhance load redistribution and improve overall stability. In contrast, the 11x11 frame, characterized by a highly fragmented and less organized system, shows a markedly lower ultimate load, indicating a reduced ability to sustain stress without experiencing local or global failure. However, the more critical aspect of the analysis emerges when damage is introduced into the system. The study simulates progressive collapse scenarios by removing key structural elements, such as beams and columns, within a predefined damage area. The response of each frame to this localized damage reveals important trends regarding the ductile-to-brittle transition at the structural scale. The results indicate that the 2x2 frame, despite having the highest initial strength, experiences a sharper decline in once damage extends beyond a critical threshold. The reduction in load-bearing capacity is less abrupt in the 5x5 and 11x11 frames, suggesting that while hierarchical structures provide superior resistance in the intact state, their transition to brittle failure is more pronounced when subjected to significant damage.

A more detailed analysis of the results reveals a key threshold value for ductile-to-brittle transition, which occurs when the width of the damage zone reaches approximately half the inter-story height. This threshold is visually evident in the graphical representation, where the fragility of the 2x2 frame increases rapidly beyond this point, eventually converging with the fragility values of the 5x5 and 11x11 frames. This suggests that hierarchical structures, while more robust under minor damage conditions, become just as vulnerable as less organized systems once damage extends beyond a critical size. This behavior closely mirrors fracture mechanics principles in materials, where the presence of small cracks may not immediately lead to failure, but once a critical crack length is exceeded, brittle fracture becomes inevitable.

The study further investigates the fragility number, introduced as an analogy to fracture mechanics parameters used to quantify material brittleness. This number is derived using the stress intensification factor and provides a measure of how susceptible a structure is to brittle collapse. The results indicate that the fragility number is lowest in the 2x2 frame, reinforcing its more ductile response under localized damage. However, as the damage width increases, the fragility number of the 2x2 frame rises more steeply compared to the other configurations. This finding suggests that while hierarchical structures offer superior resistance to initial damage, they reach a critical fragility threshold more quickly than less organized systems. This pattern is consistent with the fracture

behavior of materials, where larger specimens with higher toughness exhibit delayed crack initiation but undergo sudden failure once a certain stress intensity level is exceeded. The findings suggest that although increasing the number of structural elements can enhance redundancy, excessive fragmentation leads to reduced load redistribution capabilities, making the structure more susceptible to sudden collapse.

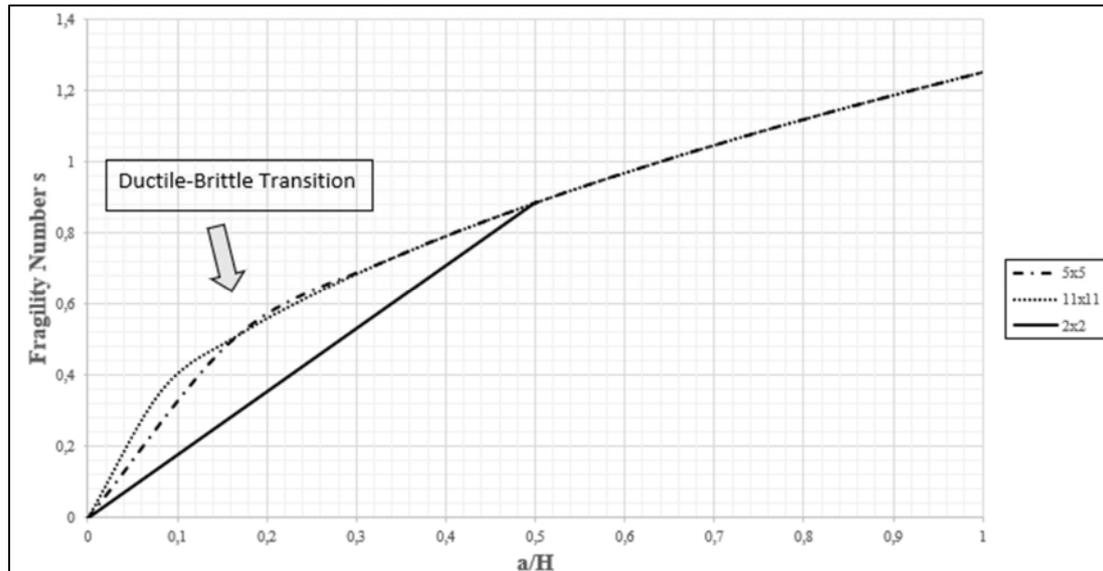


Figure 3 Transition from Ductile to Brittle behaviour in studied frames with damage CB

In the context of framed structures, this suggests that highly heterogeneous systems lack sufficient internal load redistribution mechanisms, making them susceptible to premature brittle failure. This deviation also highlights an important design consideration: while increasing the number of structural elements may initially seem beneficial for redundancy, excessive fragmentation without sufficient hierarchy can make a structure more fragile rather than more robust. The results indicate that a threshold exists beyond which reducing structural hierarchy leads to a disproportionate increase in fragility, making progressive collapse more likely even under minor damage scenarios.

By extrapolating these findings to real-world structural applications, the study provides valuable insights into the design strategies that can enhance robustness and mitigate progressive collapse risks. The results emphasize the importance of balancing redundancy and load redistribution capacity to optimize structural performance. One key implication is that while increasing hierarchy improves initial strength, engineers must also consider the consequences of damage propagation and ensure that failure mechanisms remain controlled and gradual rather than sudden and catastrophic. These findings align with existing research on earthquake-resistant design, where ductility-enhancing measures such as confinement reinforcement and energy dissipation devices are implemented to prevent abrupt failure.

The study also highlights the applicability of fracture mechanics principles at the structural scale, reinforcing the idea that material behavior and structural performance are governed by similar underlying mechanisms. Just as crack propagation in brittle materials follows well-defined stress intensity thresholds, the failure of framed structures under progressive damage can be analyzed using equivalent concepts. This opens new possibilities for applying fracture mechanics-based design methodologies to large-scale engineering projects, improving the predictability of structural behavior under extreme conditions.

Ultimately, the results of this study confirm that structural hierarchy is a double-edged sword in engineering design. While higher hierarchy enhances initial robustness and increases resistance to localized failures, it also introduces a critical transition point where the structure's fragility escalates rapidly. The identification of this threshold provides engineers with a valuable tool for optimizing structural layouts, allowing for better-informed decisions in designing buildings and infrastructures that can withstand progressive collapse scenarios. By integrating these insights into modern engineering practices, it becomes possible to develop safer and more resilient structures, capable of maintaining stability even in the face of significant damage or unexpected failure events.

## 4 Implications for Structural Design and Engineering

The findings of this study have significant implications for structural design, particularly in the context of progressive collapse prevention. While increasing hierarchy improves initial strength and enhances robustness under small damage conditions, it also introduces a critical transition threshold where fragility escalates rapidly once damage extends beyond a certain limit. This suggests that designers must carefully balance the trade-off between redundancy and hierarchy to optimize structural resilience. The results also reinforce the applicability of fracture mechanics principles at the structural scale. Just as cracks in brittle materials dictate fracture behavior, localized damage in framed structures determines whether failure occurs gradually or abruptly. By integrating these insights into engineering practices, new methodologies can be developed to predict and mitigate the risks of brittle failure, ensuring safer and more resilient infrastructures.

## 5 Conclusions

This study establishes a novel analogy between material fracture mechanics and structural collapse in framed buildings, demonstrating how classical fracture mechanics principles can be extended to predict and analyze progressive collapse mechanisms. The results confirm that structural hierarchy plays a dual role in failure resistance: while it enhances initial robustness, it also introduces a threshold beyond which brittle failure becomes inevitable. The deviation of the  $\sigma$ — $\epsilon$  fragility curve for small damage levels highlights how reduced hierarchy can lead to brittle collapse even under minor disturbances. These findings emphasize the need for design strategies that maintain an optimal balance between ductility and redundancy, ensuring that structures can sustain localized damage without triggering sudden collapse. By leveraging fracture mechanics concepts in structural design, engineers can enhance predictive modeling capabilities and develop safer, more resilient buildings.

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