Iterative and multilevel methods for PDE-constrained optimization under uncertainty

Fabio Nobile*, Tommaso Vanzan†

* Institute of Mathematics
Ecole Polytechnique Fédérale de Lausanne (EPFL)
Station 8, 1015 Lausanne, Switzerland
e-mail: fabio.nobile@epfl.ch, web page: https://www.epfl.ch/labs/csqi

† Dipartimento di Scienze Matematiche
Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129, Torino, Italy
e-mail: tommaso.vanzan@polito.it

ABSTRACT

In this talk we consider Optimal Control Problems (OCPs) constrained by random Partial Differential Equations (PDEs), where an optimal (deterministic) forcing term is sought so as to minimize the expected value or other risk measures of a cost functional. We will first focus on iterative methods to solve the optimality system associated to the OCP once the underlying PDE and the probability space have been suitably discretized. Such optimality system is typically of very high dimension, characterized by a coupled system of N state PDEs, N adjoint PDEs, and a single optimality condition, where N is the number of collocation points used to discretize the probability space.

We present few preconditioning and multigrid strategies to solve efficiently the coupled optimality system and test them on several cases, including an OCP with box constraints and L^1 penalization on the control, and a risk-adverse OCP involving the smoothed CVaR risk measure.

We then discuss how multilevel ideas and hierarchical approximations can be used in the the context of PDE-constrained OCPs under uncertainty adopting a combination technique approach, which requires solving the OCP for several low-fidelity discretizations of the PDE and quadrature formulae to compute the expected cost functional. All the computed solutions are then linearly combined to get a final approximation. We show some theoretical error estimates and present numerical results showing the efficacy of the approach.

REFERENCES

[1] F. Nobile and T. Vanzan, Multilevel quadrature formulae for the optimal control of random PDEs, July 2024, Available as arXiv:2407.06678
[2] F. Nobile and T. Vanzan, A combination technique for optimal control problems constrained by random PDEs, SIAM J. Uncert. Quant. 12(2), p. 693–721, 2024. Doi: 10.1137/22M1532263
[3] G. Ciaramella, F. Nobile and T. Vanzan, A multigrid solver for PDE-constrained optimization with uncertain inputs, J. Sci Comput, 101, article 13, 2024. Doi: 10.1007/s10915-024-02646-7
[4] F. Nobile and T. Vanzan, Preconditioners for robust optimal control problems under uncertainty, Num. Lin. Algebra with Appls. 30(2), p. e2472, 2023. Doi: 10.1002/nla.2472S.