STRUCTURAL ANALYSIS OF HISTORICAL CONSTRUCTIONS II
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ANALYSIS OF THE STRUCTURE OF GOTHIC CATHEDRALS APPLICATION TO BARCELONA CATHEDRAL

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SUMMARY

The study of ancient structures composed of stone arches and vaults requires the use of specific techniques of analysis endowed with powerful procedures for the modelling of geometry, the simulation of the mechanical response of the various materials (including ashlar blocks and masonry, backings and fills) and the simulation of the possible agents (gravity, wind, earthquake, settlements) which may affect the structure or may have affected it through history. More specifically, those techniques must be able to reproduce the actual conditions of thrust equilibrium between the large number of curved, linear or two-dimensional members involved.

The validity of several modelling techniques with a very different level of sophistication is studied through their use in the study of a particular Gothic construction: Barcelona Cathedral. The various techniques used were developed at the School of Civil Engineering of the Technical University of Catalonia and have already been used in the analysis of a set of historical constructions, including the Basilica of San Marco (Oñate et al., 1997) and the Crypt of the Colònia Güell (Roca, 1997).

The present study of Barcelona Cathedral, now under development, is thus not only aimed at gaining a better understanding of various aspects of the resistance of Gothic constructions (and the southern and Catalan Gothic, in particular) but also at appraising the possibilities and limitations of various analytical techniques.

1 SOME CONSIDERATIONS REGARDING GOTHIC STRUCTURE

1.1 Overall structural design

The resistance of Gothic construction gave rise to a certain amount of controversy after Viollet-le-Duc's first serious attempt to propose a rational vision of its mechanical principles and particular structural ordering. His enthusiastic interpretation of Gothic structure as a fully rational object was later contradicted by other researchers (see B. Bassegoda, 1974). Rather than defending Viollet's points of view, they found instances of structural arbitrariness and uncertainties which were only to be understood as compositional motifs, construction devices, or purely a consequence of the limited knowledge or intuition of the historical designers.

Viollet's theory presented Gothic structural ordering as a system strictly based on equilibrium and justified by it. Furthermore, all devices of Gothic structure were laid out so as to contribute to the achievement of an overall ductility. In this context, ductility must be understood as the capacity of the structure to accept significant deformation without sustaining damage or failing. It is this ductility which allowed the Gothic structural components (piers, arches, vaults) to attain their impressive dimensions and their slenderness.

More recently, other researchers have found notable structural weaknesses in Gothic constructions. Apropos of the study of the Cathedral of Sta. María, in Vitoria, Spain, Croci et al. (1995) observed weak points such as (1) a very limited interconnection between the structural elements, producing a type of isostatic construction; (2) the excessive slenderness of piers and related possible influence of second-order effects in the equilibrium, (3) the deformability of buttresses and flying arches, (4) the eccentricity created over the pier by buttresses attached to pier extensions which usually do not stand directly over the pier, (5) the weakening effect caused on support elements by the triforium and possible openings, with special regard to their shear resistance, and (6) the fact that rib springings in the aisle and arcade arches may strongly reduce the local resistant sections of the upper pier extensions. The combination of these aspects are the cause of major deformations and dangerous out-of-plumbness. According to the researchers small design or construction imperfections may lead to high deformability and cracking.

Furthermore, frequent damage patterns have been identified which in some cases are considered "chronic", such as the so-called Sabouret cracks typically observed in large groined vaults (Heyman, 1983, Barthel, 1993).

The study of several major Gothic constructions, including the case of the Basilica of St. Francis of Assisi after the September 1997 earthquake (Croci, 1998), also showed the weakness of Gothic structures to special agents such as earthquake, intense wind and large foundation settlements.

1.2 The role of the structural elements

The clear and univocal role that Viollet-de-Duc assigned to the various structural elements (diagonal, transverse and clerestory arches in vaulting, buttresses and flying buttresses, etc.) and the very assumption that all these elements are essential in the Gothic grammar, are also today disregarded by some specialists. Some new understanding came from the observation of the states of residual equilibrium attained by Gothic constructions after partial destruction caused by fire, bomb explosions (during World War II) or simply lack of maintenance. Remarkably, some vaults maintained a perfect equilibrium after losing the transverse or diagonal ribs. Other constructions lost some buttresses or flying buttresses without immediate damage being observed in the system of vaults (B. Bassegoda, 1974).

With regard to the role of the elements in a cross vault, some studies revealed that the diagonal, longitudinal and transverse elements contribute little, if anything, to resistance. The stability of the vault is due mainly to the capacity of the webbing of the vaults to work as a shell with double curvature (Mark, 1982, 1993). It is interesting to note that this contradicts some classical opinions which assigned to the diagonal arches the main structural role and relegated the webbing to little more than a simple closure.

The role of the fill placed over the vaults is also subject to opinion and probably has very different functions depending on its origin and material composition. The backing - placed at the haunches of the vaults - is of utmost importance to stabilise the vaults subject to gravity load. Additional fill placed over the vault might also produce a marginal stabilising effect when the vault is only subject to gravity loads. However, such a fill may have very different effects in the case of agents other than gravity. The studies carried out on the Basilica of St. Francis of Assisi after the earthquake showed the non-cohesive filling existing over the vaults, which was not originally placed there on purpose but merely accumulated over the decades and had a very unfavourable effect during the occurrence of the earthquake leading to the collapse of some of the vaults (Croci, 1988).

The fill which exists in some southern Gothic constructions (such as in Sta. Maria del Mar in Barcelona and Barcelona Cathedral itself) consists of a cohesive mass of lime mortar and traditional ceramic vessels and is thus characterised by considerable strength combined with very limited weight (J. Bassegoda, 1983). Supposedly, this kind of concrete fill makes a significant contribution to resistance in the cases mentioned, where a vault webbing much thinner than in most Gothic constructions has also been observed.

2 TECHNIQUES APPLIED TO THE ANALYSIS OF GOTHIC STRUCTURE

2.1 Difficulties of the analysis

Because of the many difficulties which are encountered in the study of Gothic constructions, few scientific studies have been performed to date with a view to gaining a deeper understanding of their mechanical and resistance properties. The main difficulties are posed by the following aspects:

Because of the fragile nature of stone masonry, analytical studies based on the hypothesis of linear elasticity are of very limited validity. In fact, without a great deal of further interpretation by the analyst, they may only inform about the service behaviour provided that stresses keep within moderate values.

The knowledge of both the internal composition of the structural members and the mechanical properties of the existing materials are normally very limited, for several reasons. Furthermore, both the composition and the material properties may show considerable scattering throughout the entire construction.

The mechanical behaviour is deeply dependent on the historical construction process, which is mostly uncertain. Additional uncertainty is provided by contingencies such as accidents and partial rebuilding, during construction or immediate remedial measures to constrain precocious deformation.

Finally, as we are reminded by Cassinello (1998), those constructions which have reached the present day have usually undergone major transformations as a consequence of progressive or permanent deformation, cracking, physical or chemical degradation of materials, or possible construction alterations or replacements. Again according to Cassinello, the elaboration of an accurate model should thus take into account the three main aspects determining the structural behaviour, namely the (deformed) geometry, internal composition, and material properties. The integration of the historical transformations should be extended to these three aspects.

2.2 Significant studies and developments

A widespread set of techniques has been used in the study of Gothic construction, including photoelastic modelling, classical theories based on plastic theorems, conventional matrix calculation and the finite element method.

Photoelastic modelling

The research carried out by Mark (1982) by means of photoelastic models, extended to a large number of major cases such as the cathedrals of Amiens, Beauvais, Bourges, Chartres and Palma de Mallorca, constitutes one of the first and more successful attempts to quantitatively analyse the structure of these constructions to gain a deep understanding of their performance when subject to various agents (wind in particular). The study enables us to envisage the essential characteristics of the buildings, compare their different design, and better understand the role of their different structural elements. It also provides an understanding of the origin of some of the existing damage, such as certain cracks and permanent deformations.

Plastic analysis

The possibilities of classical plastic analysis in the study of Gothic cathedrals, based on the plastic limit theorems, have been shown by Heyman (1998). Due to the validity of the hypothesis assumed by classic theory, very realistic collapsing mechanisms may be devised through its careful application. However, classical analysis is of little use if applied to conditions other than or far removed from failure. Thus, classical theories are not very useful to interpret the cause and extent of cracks, deformation or other damage not directly related to the generation of a collapse. Furthermore, their practical but rigorous use becomes extremely difficult in the case of complex structures with multiple elements (as encountered by several authors, including Cauvin and Stagnitto, 1998).

According to Chassagnou et al. (1998), the evaluations obtained using classical theories tend to be too pessimist and may lead to repairs that are more far-reaching than actually needed. This is so because of the type of additional hypothesis that must be assumed to make the use of these theories feasible.

Conventional matrix calculation

Using conventional matrix formulation, Leon and Cassati (1997, 1998) developed a detailed parametrical study of the main geometrical and material variables influencing the structural response of León Cathedral in Spain. The study was based on a careful modelling of the main transverse section of the building and the nave vaults by means of an equivalent system of one-dimensional elements.

Finite element method

The main limitation of most finite element formulations, when applied to masonry elements, lies in the hypothesis of the continuum, which makes it difficult or even impossible to simulate the particular kinematics of the masonry modes of failure. Elastic analyses based on the finite element method are only suitable for characterising working conditions subject to states of very moderate stresses in which compressive stresses are largely predominant. However, the adoption of suitable constitutive equations which account for the main phenomena related to the failure of the materials (cracking under tension, crushing under compression, etc.) makes it possible to reproduce more advanced stages of the response and even to simulate failure mechanisms similar to those predicted by the classical theories.

Croci et al. (1995) carried out a finite element analysis of the Cathedral of Sta. María, in Vitoria, Spain. The analysis, applied to the main transverse sections of the building and to the nave vaults, followed an incremental strategy to account for cracking due to tension or shear stresses, as well as the equilibrium second-order effects. This analysis showed some of the main weaknesses of the building, as has already been mentioned in Section 2.1.

Similar analyses were also used for the study of the collapse of Beauvais Cathedral (Croci et al., 1998b) and the effects of the earthquake of September 1997 on the Basilica of Assisi (Croci, 1998).

The finite element method, also combined with suitable constitutive equations, has been successfully used to study Gothic cross vaults (Barthel, 1993). For this purpose, very detailed finite element models were elaborated and used in combination with a material treatment enabling the simulation of cracking as well as sliding between arch ring joints.

Cauvin and Stagnito (1993, 1995) carried out very interesting studies using both classical plastic analysis and non-linear analysis by the finite element method. Their method was successfully applied to analyse the central nave of Reims Cathedral.

The finite element method has been also utilised to study several Spanish Gothic cathedrals (Burgos, Seville) by Izquierdo (1997; see also Cassinello, 1998).

Chassagnou et al. (1998) have developed a technique for the generic modelling of Gothic cross vaults, also based on the finite element method, with the aim of making available a tool for the practical evaluation of their structural response.

Oñate et al (1997) have developed a finite element damage model which was successfully used for the analysis of St. Marks Basilica in Venice. This model, briefly discussed in a later section, is one of the techniques chosen for the numerical studies of the Cathedral of Barcelona presented in this paper.

3 BARCELONA CATHEDRAL

3.1 Introduction to the study

The choice of Barcelona Cathedral (Figs. 1-6) for the purpose of the present study is partially due to the fact that abundant information exists on its geometry and other construction aspects, made available to the author by the Diocesan Archive and the Chapter of Barcelona Cathedral. Furthermore, there are some publications with interesting historical and construction information (Elías, 1926, J. Bassegoda, 1968, 1981, 1983). Barcelona Cathedral also shows certain structural singularities which gave it added interest as a subject of study.

- First, it is a construction in the so-called Catalan Gothic style, which exhibits notable specific features. Catalan Gothic constructions were typically designed as very diaphanous spaces with large-spanned naves.

- The dimensions of the vaults are remarkable due to the large gap between the piers not only in their transversal direction, but also longitudinally.

- The lay-out of the transverse section of the building is characterised by the aisle naves being almost (but not quite) as high as the central nave. Because of this structural lay-out, the aisle vaults have an important structural function in retaining the thrust of the central vaults.

In a celebrated lecture on Mallorca Cathedral, Rubió i Bellver (1912) distinguished between three different types of Gothic cathedrals with regard to the structural organisation of their transverse section. In the first type, the nave and the aisles are built to the same height (as in Saragossa, Munich, Perugia and others). The second type, in which the aisles are almost but not quite as high as the nave, is by far the least widespread since it only includes the cases of Barcelona Cathedral and Sta. María del Mar. The third type, in which the aisles are significantly lower than the nave, is by far the most frequent (being the case in Amiens, Reims, Beauvais, Cologne, Milan, Toledo, Mallorca and many others).

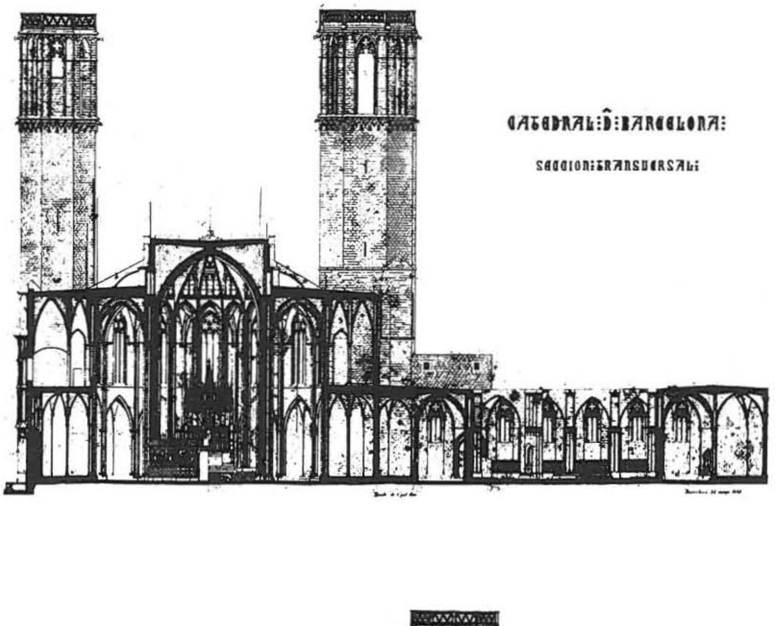

Fig. 1 Longitudinal and transverse sections of Barcelona Cathedral (facsimile reproduction of plans drawn in 1864 as part of the studies previous to the construction of the new cimborio).

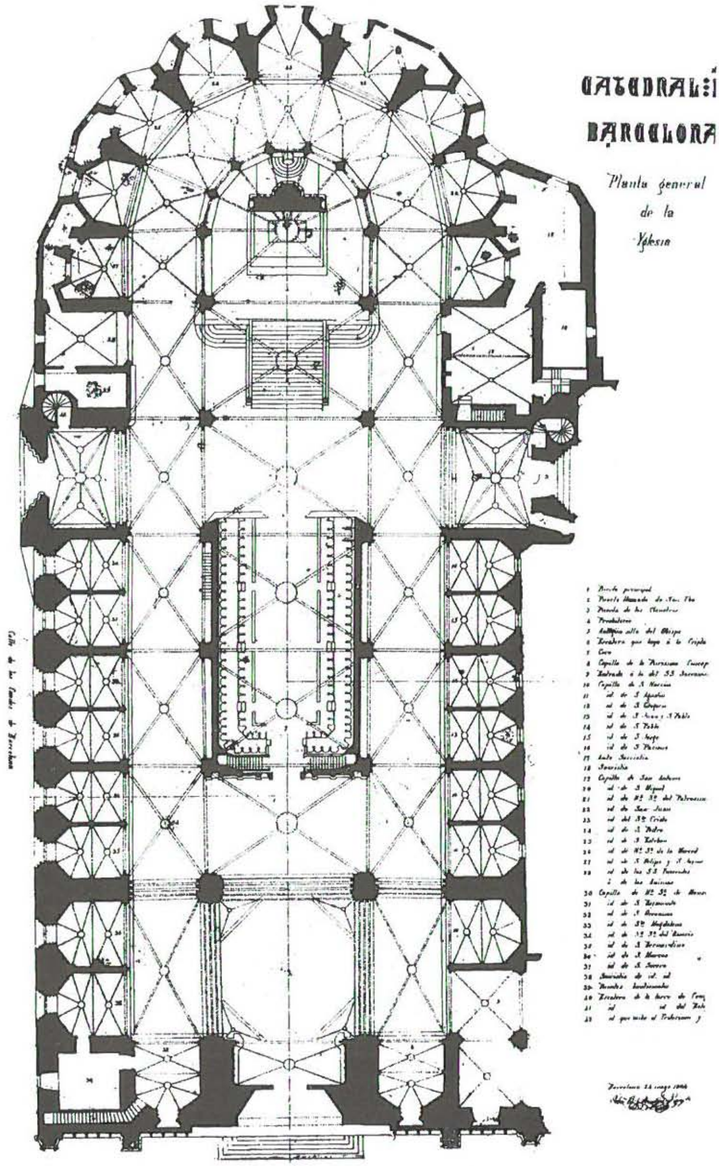

Fig. 2 Plan of the building (1864 plans).

Following Rubió, in the case of constructions belonging to the second type the thrust of the nave vaults is completely retained by the aisle vaults so that no flying arches are needed at all. Where they exist, as in Barcelona Cathedral, they are non-structural and their only function is probably drainage.

The prominent role of the aisle vaults as stabilising elements may be related to two construction devices which are consistent with them and which were probably incorporated in order to strengthen them. In particular, the aisle vaults are surcharged with an amount of concrete fill greater than that placed over the nave, as if to increase their counteracting thrust.

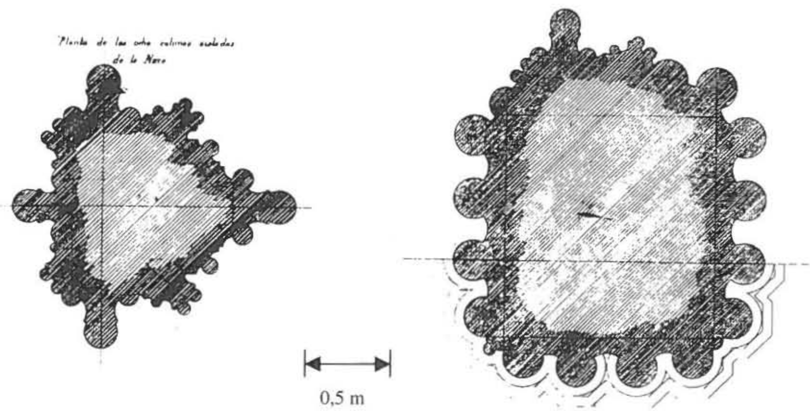

Fig. 3 Transverse section of the nave piers and the piers sustaining the cimborio as represented in the 1864 plans (*E 1:40*).

3.2 Brief description of the building

Construction of the naves of Barcelona Cathedral was begun in 1298 and lasted for more than a century. As usual, the apse was constructed first, being finished in 1327, while the construction of the entire nave continued until 1417. In 1422 work stopped, leaving the cimborio unfinished and a provisional wall closure as a façade. Most of the original features of the building are assumed to be attributable to the master builder Jaume Fabre, who worked on the Cathedral from 1317.

The plan of the temple shows a very unconventional lay-out in which the cimborio is not erected over the crossing but over the first bay of the nave close to the façade, while the crossing is delimited by the two majestic clock towers, more commonly part of the façade. A similar distribution can be observed in very few cases, such as the German cathedrals of Ulm and Friburg.

The building includes three naves (the nave and two aisles) although, as a consequence of its particular design, it appears to enclose two additional aisles. This particular effect is caused by the inclusion of the buttresses in the interior space and their use to laterally confine the side chapels. The space between the buttresses is vaulted at an intermediate level, to form a tribune across the side aisles. The upper vaulting between the buttresses stands at the same height as the aisle vaulting and gives the impression of being an extension of the former (Fig. 4,5).

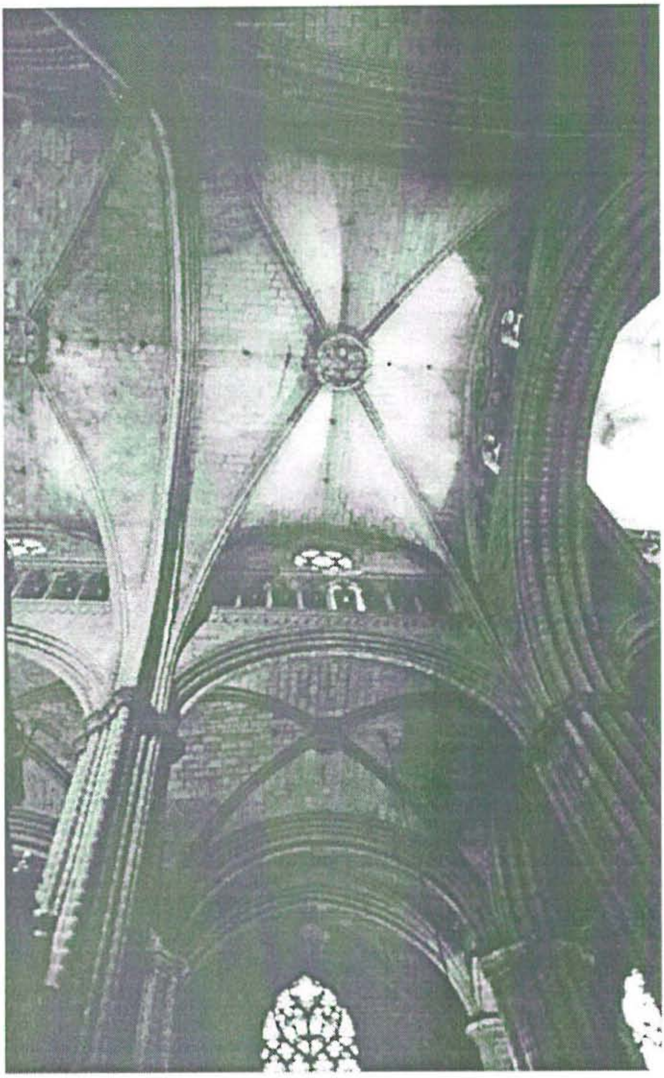

Fig. 4 View of the nave vault at the bay close to the cimborio.

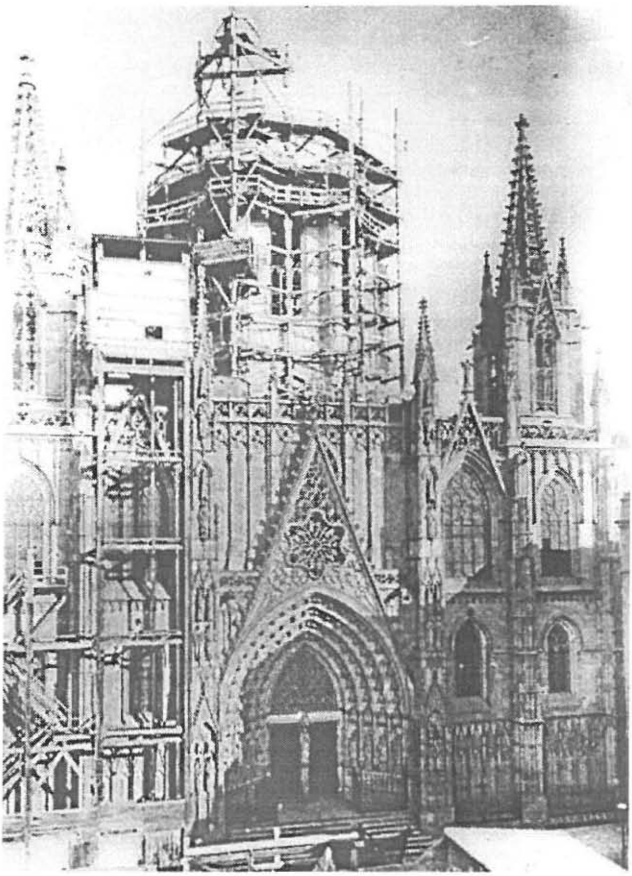

Fig. 5 View of the façade during the construction of the new cimborio (circa 1900).

The main dimensions of the building are 93x45m. The main transverse inner width, including the side chapels, is 37.3 m, while the joint width of the central nave and aisles is 25.6 m. The nave spans 12.80 m and has a maximum high of 25.6 m. The span of the side aisles is equal to one half the span of the nave. The rise of the vaults at the side aisles, of 20.5 m, begins close to the springings of the central vaults.

The triforium, usually placed over the arcade, is here disposed above the springings of the main arches of the clerestory, while the remaining space between the triforium and the clerestory arches is used to inscribe a series of oculi. The resulting disposition contributes to enhance the feeling of height in spite of the rather modest vertical dimension of the building.

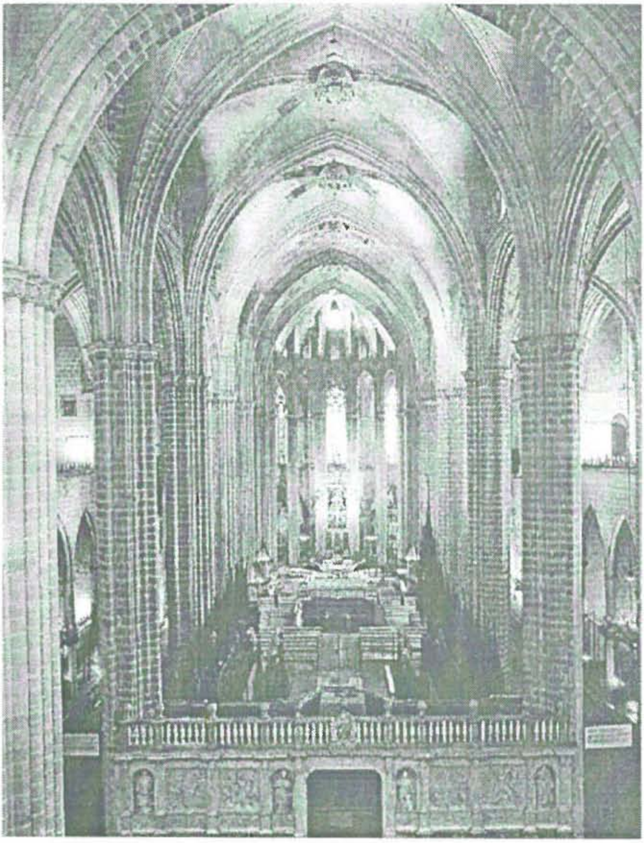

Fig. 6 General view of the nave.

It should be noted, notwithstanding, that Barcelona Cathedral shows an architectural conception different from that of the more emblematic examples of northern Gothic. The typical corridor lay-out of the northern cathedrals is here avoided, and thus a more regular and diaphanous space is produced. To some extent, it is the Roman or Palaeo-Christian basilica, rather than the northern Gothic tradition, that seems to be inspiring this particular new conception. Barcelona Cathedral constituted an innovative construction in which the specific devices of Gothic architecture were efficiently adapted to the climate and cultural idiosyncrasy of its southern context. The further development of these new dispositions that were to lead to the conception of very diaphanous constructions such as the Basilica of Sta. Maria del Mar, also in Barcelona, or, later in history, Mallorca Cathedral, which is considered by some specialists as the epitome of all Gothic architecture (Rubió, 1902, Mark 1982).

4 ANALYSES

4.1 Lay-out of the analysis of Barcelona Cathedral

Two different techniques, characterised by their very different degree of sophistication, were tried in order to carry out the analysis of Barcelona Cathedral. First, a particular formulation for the non-linear analysis of spatial framed structures with curved members was used as an initial, more economic approach. Second, a more accurate analysis was carried out using a finite element model using three-dimensional solid elements, in combination with a sophisticated constitutive equation of damage.

The two techniques were tried for two different purposes. The first was the analysis of an isolated nave vault, in an attempt to gain a better understanding of its mechanical behaviour and the role of the various elements which compose it (in particular, the role of the concrete fill). Secondly, a typical bay was analysed in order to understand the overall equilibrium and capacity of the structure subject to dead load.

The GMF model is also used to study the response of the typical bay subject to the effect of an earthquake acting transverse to the axis of the nave.

Non-linear formulation for 3D framed structures with curved members (GMF)

The adopted frame formulation consists of an extension of a previous development by Marí (1983) for concrete spatial structures to the geometric and material non-linear analysis of masonry skeletal constructions. Consistently with the principles of matrix methods, the flexibility formulation stems exclusively from equilibrium between external and internal forces at any point within an arch or linear member, so that no additional hypotheses over the displacement or stress field are required. Since the movements are fully free (unlike in FEM, where field displacement shapes must be assumed), arbitrarily high concentrated curvatures associated with damage can be reproduced, resulting in a feasible approach for damage localisation, or hinge formation. For the same reason, the numerical problems that are encountered when a perfectly brittle constitutive equation is used in combination with the displacement formulation of FEM do no manifest themselves in this case.

In order to carry out the non-linear material analysis, masonry is treated as a linear elastic-perfectly brittle material under tension, while elasto-plastic equations are adopted for masonry subject to compression and shear. A Mohr-Coulomb failure envelope is adopted in order to describe failure modes due to combined states of compression and shear.

The algebraic lay-out of this formulation, together with aspects relating to its numerical implementation and some applications to ancient structures, have already been described in several publications (Molins et al., 1995, 1998).

3D Finite Element Damage Model

Recent investigations (Cervera et al., 1990, Cervera et al, 1992, Faria and Oliver, 1993) support the idea that the non linear behaviour of concrete and masonry can be modelled using concepts of damage theory only provided an adequate damage function is defined for taking into account the different response of concrete under tension and compression states. Cracking can, therefore, be interpreted as a *local damage effect*, defined by the evolution of known material parameters and by one or several functions which control the onset and evolution of damage.

One of the advantages of such a damage model is the independence of the analysis with respect to cracking directions which can be simply identified *a posteriori* as the locus of damage points once the non-linear solution is obtained. This allows to overcome the problems associated to most elastic-plastic-brittle smeared cracking models. In this work a model developed in recent years by the authors group for non-linear analysis of concrete based on the concepts of *damage* above mentioned has been chosen. The model can take into account all the important aspects which should be considered in the non-linear analysis of concrete and masonry structures such as the different response under tension and compression, the effect of stiffness degradation due to mechanical and physical-chemical-biological effects and the problem of objectivity of the results with respect to the finite element mesh.

In essence damage models directly reproduce the loss of stiffness associated to microcracking. A damage (fractured) point is assumed to have a reduction in stiffness which is proportional to the average number of defects (voids) in a micro-region. The simplest damage model chosen here defines a single damage function ranging from zero, for undamaged intact material, and one (in fact an unreachable bound), for the complete loss of resistance at microstructural level. The loss of stiffness at each material point is then assumed to be proportional to the damaged parameter which evolution from zero to one is adequately characterised by an experimental law defined via experimental testing.

Damage models are particularly useful to evaluate the loss of stiffness at both local and global structural levels. For the later purpose a global damage parameter can be defined in terms of the strain energy for damaged and undamaged states (Hanganu et al. 1997). Global structural failure corresponds to a value of the global damage parameter approaching unity. This provides a useful tool for monitoring in detail the complete evolution of the non linear response of the structures up to failure.

It should be mentioned that, although the model is able to reproduce the main non-linear material phenomena of masonry fabrics, the simplest single parameter damage formulation used here does not afford the simulation of effects more specifically caused by the composite nature of the fabric, such as anisotropy due to the arrangement of the masonry.

4.2 Analysis of the nave vaults.

Study using Generalised Matrix Formulation

First, a simplified analysis was carried out by modelling the vault as an equivalent framed system composed of the diagonal, transverse and clerestory arches. The vault webbing was treated as a set of stiff links connecting the ribs and constraining their movements (Figs. 7-9). The dead load and fill surcharge were distributed on the arches according to a criterion which consisted of slicing the vault into parallel arches (Fig. 7.a), while the stiff links simulating the webbing were arranged as shown in Fig. 7.b. The mechanical properties defined for the materials were those given in Table 1.

As could be expected, this simplified attempt to study the vault proved to be inadequate to calculate the true capacity of the vaults. The non-linear analysis, carried out using the constitutive modelling described in Section 4.1, predicted the failure of the vault for an applied load smaller than the total dead load of masonry and fill, which is obviously absurd. An alternative modelling was performed by enlarging the rib sections with part of the adjacent webbing so that its contribution to the resistance could be taken into account in the working direction of the ribs. Not surprisingly, this procedure yielded better results, allowing a total load applied on the vault greater than the actual dead load. The technique could be further improved by treating the webbing as a system of diagonal elements consistent with the actual force trajectories in a Gothic vaulting (see Mark, 1982, Fig. 11, and Barthel, 1993). However, given the limited accuracy of these procedures, the use of a 3D finite element modelling seems definitely more suitable.

Fig. 7 Modelling of a vault as a system of one-dimensional elements (primary arches simulating the ribs and secondary arches simulating the webbing).

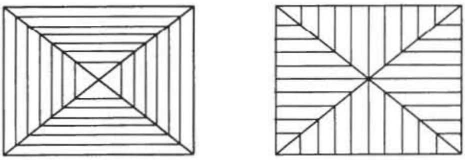

Fig. 8 Criteria for the distribution of load (a) and for the discretisation of the webbing as a set of connecting linear elements (b).

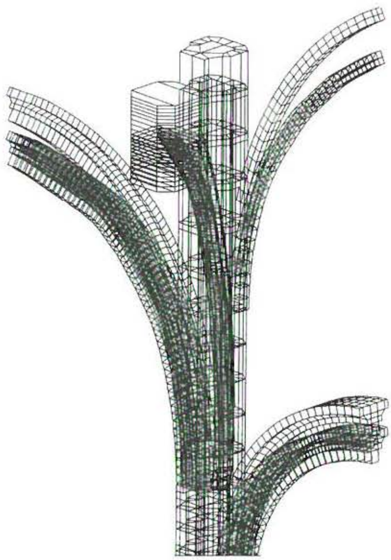

Fig 10. Deformed shape of a vault quadrant modelled as a framed system, subject to dead loading.

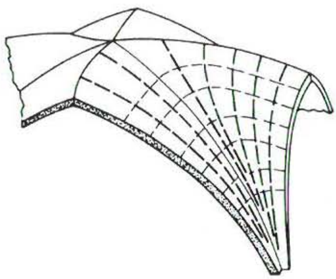

Fig. 11 Force trajectories in a Gothic vault subject to dead loading (Mark, 1982).

Analysis of the vault using 3D solid finite elements

The FEM modelling of the vault accounts for the rib system, the webbing, the rubble backing at the vault haunches, the concrete fill and the lateral walls, all defined by their corresponding mechanical properties. Given the limited information available on the material properties, some assumptions had to be made in order to assign mechanical parameters to the various materials (Table 1). The greatest source of uncertainty was the concrete fill, composed of ceramic pottery and lime mortar, the properties of which had to be roughly estimated. The ashlar masonry and rubble were estimated, on the basis of empirical criteria, from the average compressive strength (74.2 MPa) of the Montjuïc sandstone used for the construction of the building.

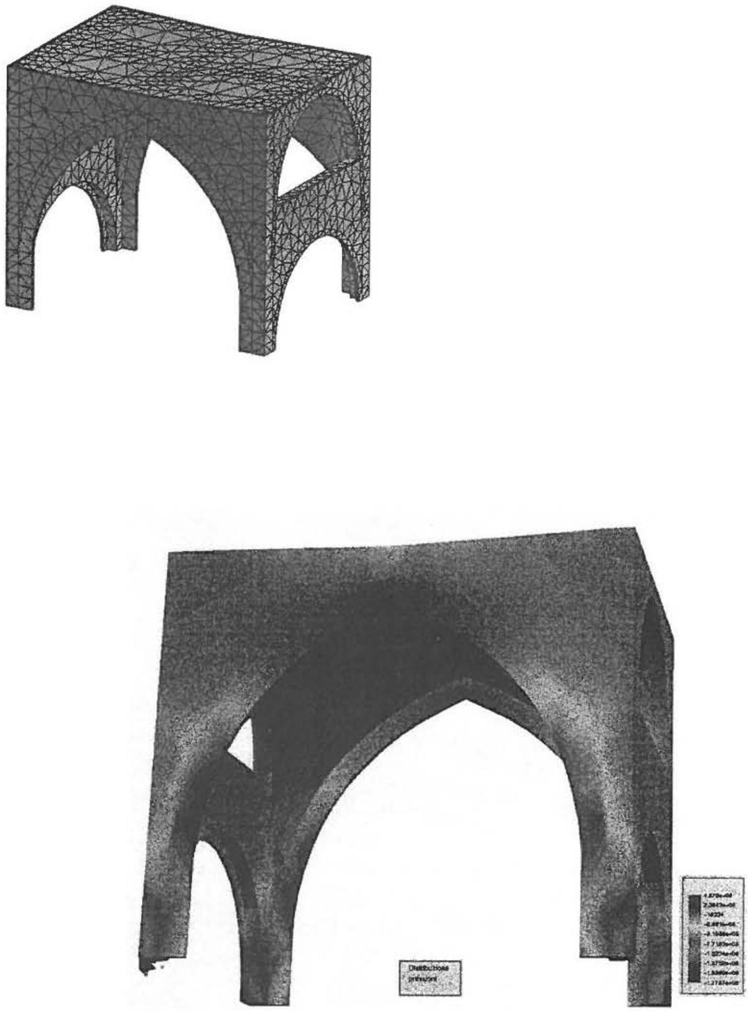

Fig. 12 FEM model and distribution of stresses caused by total dead load on the nave vault (main compressive stresses in Pa).

Fig. 12 shows the distribution of stresses which is obtained for the vault subject to total dead load. Some of the observations allowed by the simplified frame model can be confirmed, such as the significant role of the webbing in transferring the forces to the piers, and the major contribution of the transverse arches but small or null contribution of the clerestory ribs and diagonal intersections.

Table 1. Mechanical properties defined for the various materials

Material	Deformation modulus (Mpa)	Poisson coefficient	Compresive strength (kPa)	Tensile strenght (kPa)	Density (kN/m3)
Ashlar masonry (*)	8000	0.2	8000	400	2,7
Rubble (**)	2500	0.2	2500	100	2,5
Concrete fill	500	0.2	500	50	1,0

(*) in piers, ribs and vault webbing
(**) at the core of piers and at vault haunches

The analysis predicts an unrealistically high ultimate capacity (over three times the total dead loading) because the lateral displacements of the springings are defined as fully constrained in the individual treatment of the vault. Thus, the simulated ultimate mechanism does not account for the flexibility and limited capacity of the retaining system provided by the aisle vaulting and buttresses. More realistic results are obtained when, as explained in Section 4.3, the vault is inserted into a more complete model also including all structural elements in the transverse section.

4.3 Study of a nave bay

Study using Generalised Matrix Formulation

The purpose of this study was to analyse the possibility of studying the equilibrium of Gothic structure up to the ultimate condition by means of a simple model. In spite of this simplicity, the model was elaborated to obtain an accurate description of the geometry of the nave section, the distribution of the dead load and the response of the materials.

Using the faculties of the method, the various structural elements were modelled as straight or curved linear elements with variable and composite cross-section (Figs. 13-14). It should be noted that, in a very simplified way, the vaults are also treated as a curved composite beam where each material (masonry, rubble, concrete fill) is defined by its particular mechanical properties. The analysis accounts for the effects of masonry cracking under tension or crushing under compression as described in Section 4.1.

By gradually applying the gravity load up to and beyond the dead load value we were able to visualise the working condition of the structure, the progressive development of cracking and the final appearance of a number of crushed zones leading to failure (Figs. 14-16). The applied load was kept proportional to the dead load throughout the process. Failure was reached for a total applied gravity load equal to twice the total dead load.

As can be seen in the figures mentioned above, the sections affected by deep cracking or crushing develop a highly pronounced curvature and thus may be identified with hinges, which would lead to an ultimate mechanism according to the plastic theory. The set of hinges leading to the ultimate mechanism appeared in the nave vault, the upper region of the pier shafts and the aisle vaults. In both the nave and aisle vaults the hinges developed at their crown. However, the obtained mechanism is not identical to the theoretical one that would be obtained by applying the plastic theory, because of the contribution of the general deformation of the elements to the instability. Owing to this contribution, the failure mechanism becomes possible even if some hinges are not fully developed, as at the nave vault crown, or not developed at all, as occurs in the base of the piers where the mobility needed to develop the mechanism is given simply by the great flexibility of the shafts. A full theoretic mechanism would require a set of seven hinges.

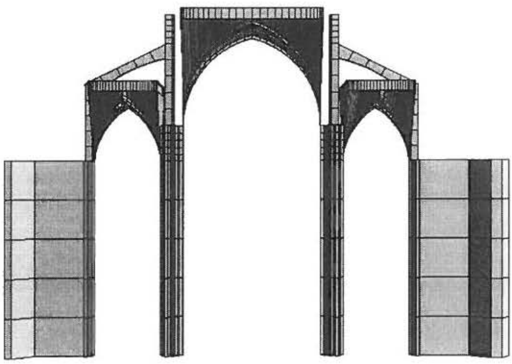

Fig. 13 Equivalent frame model.

During the process, the average compressive stress at the base of the piers varies from a value of 3.5 MPa, for dead loading applied, to a value close to the compressive strength (8 MPa).

The response is almost linear up to failure, with only very slight losses of stiffness throughout the development of the hinges and a final, fragile failure. It should be mentioned that this fragile response may result as a limitation of the numerical procedure to continue the analysis beyond some local severe damage.

The parametric studies showed that the ultimate load is not dependent upon the compressive strength when values higher than 8 MPa are considered, which means that the failure is constantly caused by the general ultimate mechanism. This is what should be expected from the classical plastic analysis. In contrast, the ultimate load decreases in proportion to the compressive strength when values of less than 8 MPa are defined because the failure is then determined by the crushing of the masonry at the base of the piers.

As demonstrated by the parametric studies, the general capacity was not significantly dependent on the stiffness of the concrete fill (for values of the modulus of deformation varying between 50 to 1000 MPa). Similarly, a moderate variation of its average density had almost no effect upon the ultimate load.

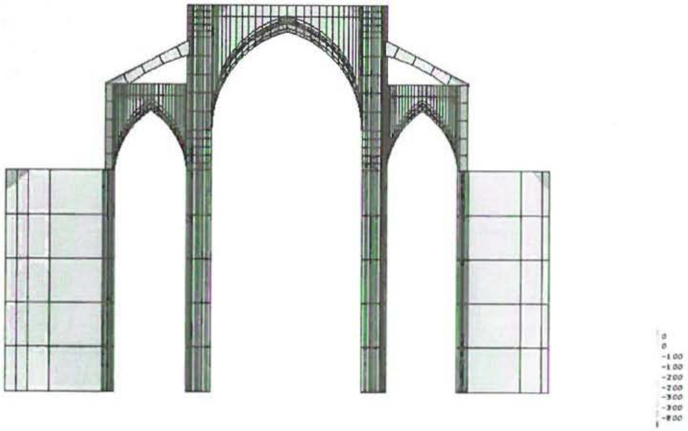

Fig. 14 Distribution of axial stresses and cracking for the structure subject to dead load.

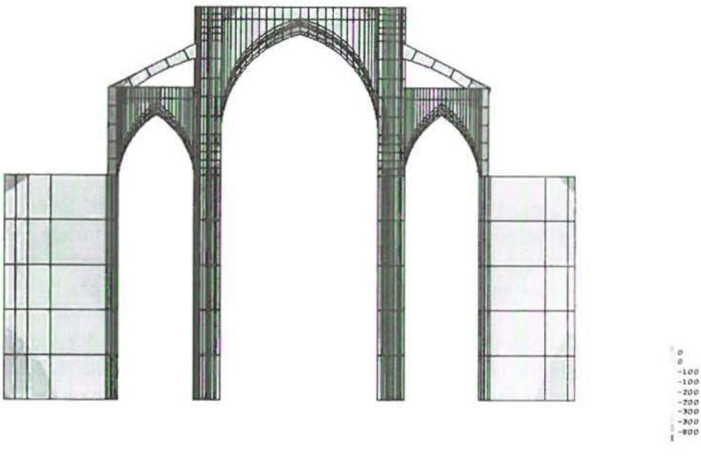

Fig. 15 Distribution of axial stresses and cracking at failure occurring for a dead load factorised by 2.

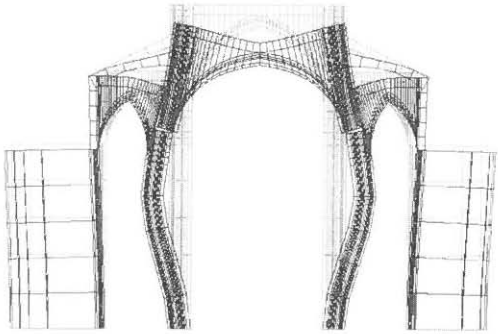

Fig. 16 Deformed shape close to failure.

This model was also used to simulate the effect of an earthquake by applying a set of horizontal equivalent static loads consistent with the distribution on mass. The non-linear analysis predicted the failure for a seismic coefficient a/g equal to 0.12, which can be considered safe enough with regard to the seismic intensity of the region of Barcelona.

The application of the Spanish seismic code would lead to the consideration of a coefficient equal to 0.086 for a masonry building in a similar condition and for a risk period of one hundred years. As shown in Fig. 17, the distribution of stresses is not significantly altered with respect to the dead load condition when a more moderate earthquake is simulated. The corresponding deformed shape is shown in Fig. 18.

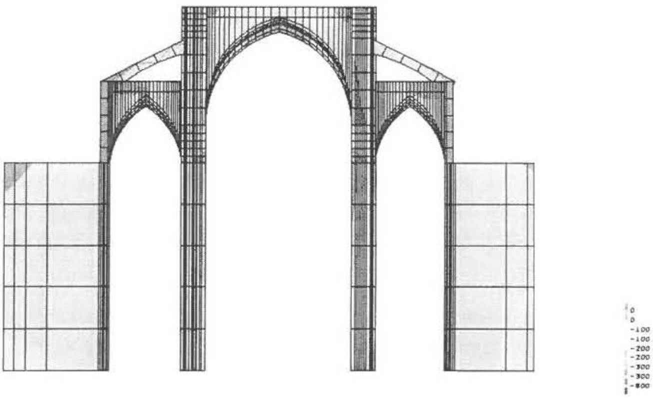

Fig. 17 Distribution of stresses caused by a set of static equivalent forces simulating an earthquake characterised by a coefficient a/g equal to 0.086.

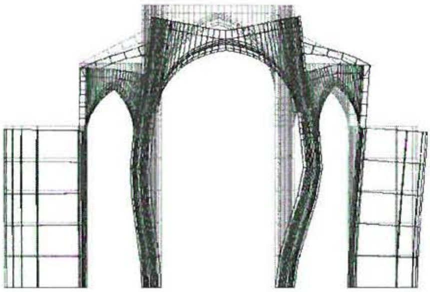

Fig. 18 Geometry deformed by earthquake.

Obviously, the limited validity of the treatment of the earthquake as an equivalent set of static forces should be taken into consideration and the obtained value only regarded as a coarse approximation to the possible seismic response of the building.

Finite element damage modelling

In order to keep the total number of equations within reasonable limits, only a quarter of the transverse section was discretised into tetrahedral solid elements to carry out the damage analysis.

Damage first appears at the crown of the transverse arches of the nave. As additional load is progressively applied, damage spreads covering larger regions of the structure (Figs. 19, 20). Additional damage focuses are observed at the crown of the aisle vaults, at the haunches of the aisle transverse arches and at the bases of the piers.

Failure occurs for a total load applied equal to twice the dead load, caused by the generation of a collapsing mechanism similar to that reproduced by the GMF model. As can be seen in Figs. 19-21, both damage and curvature are concentrated in the regions where either severe cracking or intense compressive stresses appeared in the GMF model. The main difference between the two models lies in the location of the maximum damage in the aisle vault, which in the FEM model reaches its highest intensity not at the crown but in the section closest to the vault haunch near the pier. This is also the probable location of the theoretical hinges leading to the ultimate mechanism.

As in the GMF analysis, an almost linear relationship is obtained between the amount of load and the displacements of the structure with only a very slight, gradual loss of stiffness observed with the extension of damage. This almost linear response is obtained up to the ultimate load, with no further softening branch being predicted. Also as in the GMF analysis, the fragile response might be a purely numerical effect caused by insufficient mesh refinement or by the limitations of the method used in the solution strategy.

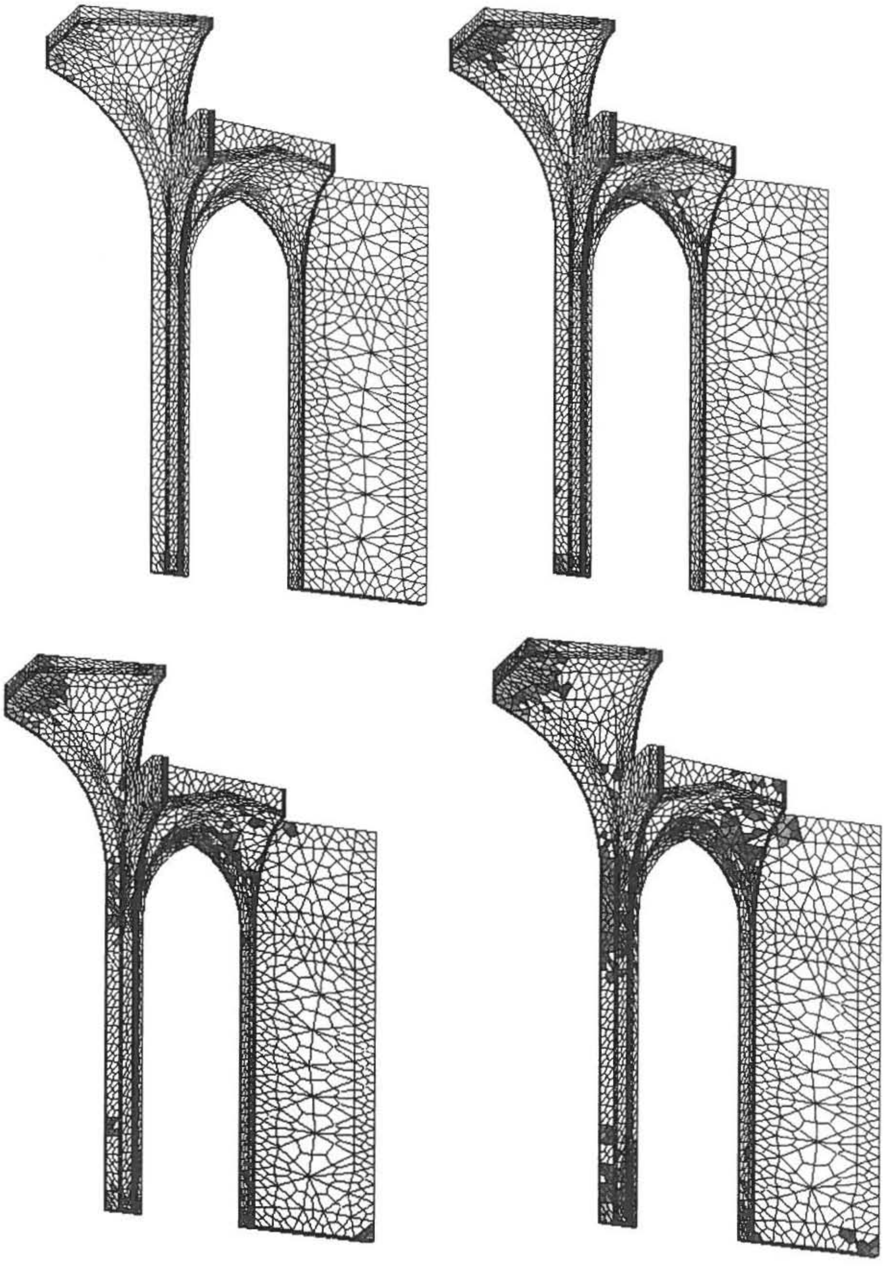

Fig. 19 Distribution of damage at loading levels corresponding to dead load factors of 0.5, 1.0, 1.5 and 2.0 (see damage scale in Fig. 20)

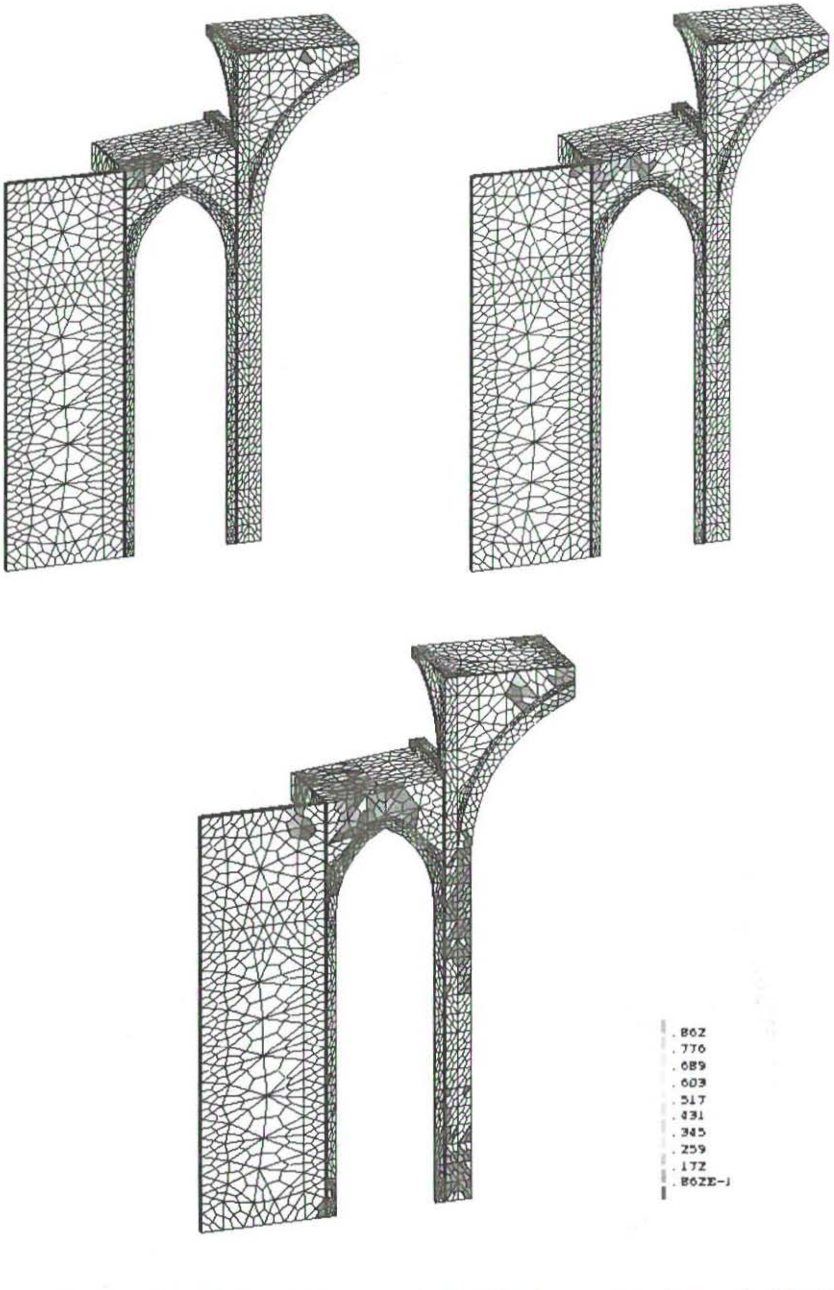

Fig. 20 Distribution of damage at loading levels corresponding to dead load factors of 0.5, 1.0 and 2.0.

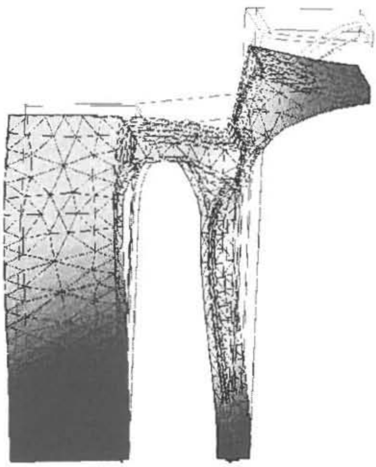

Fig. 21 Deformed shape for gravity load

5 CONCLUSIONS

Two alternate numerical approaches characterised by their very different degree of sophistication were satisfactorily used to carry out a non-linear analysis of the structure of a Gothic construction - namely, Barcelona Cathedral - taking into account the more prominent features of the mechanical response of the structural material.

The two approaches - the Generalised Matrix Formulation and the 3D finite element modelling with a damage model - produced very similar predictions for the response of the structure throughout the loading process, the collapsing mechanism and the ultimate capacity in the analysis of a nave bay.

Both approaches, but in particular the much simpler GMF procedure, owe their success in analysing masonry constructions to their ability to accurately simulate the essential factors determining their structural response. First, they share the capacity to describe complex geometry and to accurately compute and distribute dead loads on the structure. Second, they include suitable constitutive equations to model the essential characteristics of the behaviour of the material (the inability of masonry to carry tension, in particular).

The main limitation of the GMF stems from its inadequacy to treat two-dimensional elements such as Gothic vaults. If an accurate study of a vault is to be performed, the use of a detailed finite element modelling becomes unavoidable. Furthermore, it requires very realistic material modelling of the various structural elements, materials, surcharge and support conditions involved (including ribs, webbing, masonry haunches, and possible concrete fill).

Barcelona Cathedral features some notable structural idiosyncrasies, such as the stabilising role of the aisle vaulting and the contribution made to the resistance by the concrete fill over the vault. The analysis carried out provides an improved understanding of the structural significance of these peculiarities, and shows the overall structural adequacy of its almost unique design. It is because of this design that some of the frequent weaknesses of Gothic structures (as mentioned in Section 1) are not encountered here, and this is probably the case in a number of Gothic constructions. Because of the many designs or design variations, materials used, construction, maintenance and other conditions, each Gothic construction merits specific studies and conclusions, and widespread generalisation about their structural response or actual stability is not possible.

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