Hindawi
Journal of Advanced Transportation
Volume 2019, Article ID 4576961, 12 pages
https://doi.org/10.1155/2019/4576961

Research Article

Route Redundancy-Based Network Topology Measure of Metro Networks

Weiwei Jing,[1] **Xiangdong Xu**,[1] **and Yichao Pu**[2]

[1] *The Key Laboratory of Road and Traffic Engineering, Ministry of Education, Tongji University, Shanghai 201804, China*
[2] *Shanghai Shentong Metro Group, Shanghai 201103, China*

Correspondence should be addressed to Xiangdong Xu; xiangdongxu@tongji.edu.cn

Received 4 April 2019; Accepted 28 May 2019; Published 3 July 2019

Academic Editor: David Llorca

The metro system plays a very important role in the urban multimodal transportation system, yet it is susceptible to accidents. A well-designed metro system needs to provide alternative routes to travellers both in the disruptive events and the normal operating conditions for providing rerouting opportunities and balancing crowded lines. This paper provides a new dimension of assessing metro network performance—travellers' route redundancy (or route diversity), which is defined as the number of behaviourally effective routes between each origin-destination (O-D) pair in the network. The route redundancy of metro network is evaluated by statistical indicators of the distribution of the O-D-level number of effective routes. Compared with the existing connectivity and accessibility measures of topology network performance, route redundancy is also based on the topology network, but it takes the travellers' route choice into consideration. Specifically, the effective routes between each O-D pair would provide disaggregated information from the travellers' perspective. Case studies in four metropolises in the world, i.e., Shanghai, Beijing, London, and Tokyo, are conducted to examine the predisaster preparedness of the four metro networks explicitly from the perspective of route redundancy. The results indicate that the London metro network has the best route redundancy performance in terms of the statistical indicators of the distribution of the O-D level number of effective routes. Furthermore, the results of route redundancy are compared with typical measures of topology network performance in terms of measuring connectivity and accessibility of metro networks. Their differences are attributed to the fact that the route redundancy measure considers the travellers' O-D-level route choice beyond the pure network topology and the shortest path considerations of the existing measures. The route redundancy proposed in this paper could assist in evaluating the predisaster preparedness of current or planning metro networks from O-D level to network level.

1. Introduction

The metro system is becoming a priority choice to mitigate the traffic pressure in many cities, due to its promising advantages such as large capacity, high efficiency, low energy consumption, low pollution, and land resource saving [1]. As of July 2018, as summarized in Wikipedia [2–4], about 180 cities in 54 countries have opened their metro systems, and 40 cities are planning to open a metro system in the future. With the development of economy and technology, metro systems are developing towards high density, high efficiency, and networking [5]. In China, metro systems have received a rapid development in past decades, and 35 cities have opened their metro systems with a total distance of 4,898 kilometres by July 2018. For example, in Shanghai, there are 16 metro lines and 395 stations in operation with the total mileage of 673 kilometres. On the other hand, the metro ridership also continues to increase significantly. For example, in the Beijing metro system, the average daily ridership has a breakneck growth since 2008, and the maximum daily ridership has exceeded 13 million by July 2018.

Accidents (e.g., collapse, leak, terrorist attack, fire, and suicide) frequently occur in metro networks [6]. When an accident causes the failure of a station, it would affect not only the individual metro line but also multiple lines or even the whole network [7]. Since the metro network is susceptible to disruptions, measuring its performance under uncertainties has attracted a lot of attention from both researchers and practitioners. Recently, *network resilience* is being increasingly considered to be an important aspect of network performance or network behaviour following disruptions [8–10]. Resilience is often used in association with several threatening events, which may show critical and catastrophic phases such as terrorist attacks and natural disasters [11]. The White House [12] defined resilience as the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience engineering offers a much broader sociotechnical framework to cope with infrastructure threats and disruptions by focusing on three aspects: readiness and preparedness, response and adaptation, and recovery and adjustment [13].

According to Bruneau *et al.* [14], resilience can be characterized by the four "R's" concept: redundancy, robustness, resourcefulness, and rapidity. *Redundancy is one of the key dimensions of resilience* [15], which is defined as "the extent to which elements, systems, or other units of analysis exist that are substitutable". *A resilient network should be redundant as redundancy reflects the predisaster preparedness of a network*. In other words, a redundant network would provide alternative choices to reduce the impact of disruptions [16, 17]. Note that *the redundancy evaluation of the metro network is different from the vulnerability analysis*, which focuses on the consequences caused by disruptions or incidents. Interested readers are directed to the book by Taylor [10] which charts the development and a comprehensive overview of transportation network vulnerability analysis. However, *very limited attention has been paid to the redundancy of public transportation networks,* as shown by the more detailed reviews to be presented in Section 2.

This paper provides a new dimension of metro network performance assessment—travellers' route redundancy, which is measured by the distribution characteristics of the number of effective routes (or paths) between each O-D pair in a metro network. The route redundancy describes the O-D effective connections explicitly from the users' perspective. We should point out that route redundancy is still based on the topology network, but it could offer more behavioural information than the typical topology network-based measures such as the measures aggregated from the node level (e.g., degree, clustering coefficient) and the measures based on the shortest paths (e.g., diameter, network efficiency). Specifically, it can not only provide disaggregated information from O-D level, but also quantify the number of behaviourally effective alternatives considering that the travellers may not always choose the shortest paths in reality. By looking at four metro networks, i.e., Shanghai, Beijing, London, and Tokyo, we examine the performance of metro networks based on the concept of route redundancy. Furthermore, the differences and relationships between the route redundancy-based measure and the typical topology network measures will be discussed based on the four metro networks.

Redundancy reflects the predisaster preparedness of a network for combatting vulnerability. Therefore, with the networking development of metro systems, a well-designed metro network needs to provide alternative routes for travellers as much as possible under the occurrence of accidents. Also, alternative routes are needed to split the passenger flow of congested segments/lines under normal operating conditions. Hence, it is necessary to have a deep understanding of the route redundancy (or route diversity) of metro networks. Following Xu *et al.* [18], we customize the definition of route redundancy to metro networks as *the number of behaviourally effective routes (or paths) available for passengers between any two stations in the metro network*. The assessment of route redundancy could help to evaluate the predisaster preparedness of the current metro network or the planned scheme, and also offer the information of alternative routes to assist metro managers in rerouting passengers in a highly congested or disruptive event. Note that this paper only focuses on the route redundancy from the travellers' perspective, i.e., the first dimension of transportation network redundancy proposed by Xu *et al.* [18]. The second dimension (i.e., network spare capacity from the planners' perspective) of metro network redundancy will be related to the station capacity and line capacity, which itself is more complex than the first dimension. When the operation and scheduling data are available, the second dimension could also be further measured.

The reminder of this paper is organized as follows. The studies on redundancy and network topology are introduced in Section 2. In Section 3, we provide the definition and computational method of route redundancy. In Section 4, we examine the route redundancy of four metropolitan metro networks and discuss the differences and relationships between the route redundancy and the typical topology network measures. Conclusions are summarized in Section 5.

2. Literature Review

The studies on redundancy and network topology of transportation networks are reviewed in this section.

2.1. Redundancy. In a broad picture, redundancy is an important topic in systems engineering. In reality, many infrastructure and safety-critical systems require redundancy design such as the water distribution networks [19] and the aircraft door management systems [20]. The concept of redundancy has also been applied in the transportation systems as one of the measures of resilience. Berdica [21] defined redundancy in transportation systems as the existence of numerous optional routes/means of transport between origins and destinations that can result in less serious consequences in case of a disturbance in some part of the system. Immers *et al.* [22] described the redundancy of road network as sufficient spare capacity which could avoid the degradation in the quality of service. Freckleton *et al.* [23] defined redundancy as the ability of a traveller to adjust routes as necessary to detour around the affected sections of the network under disruptions. Actually, *the concept of redundancy has been widely studied but there is no consensus definition for transportation networks*, which comes down to two aspects: the number of alternatives and the network spare capacity (e.g., [10, 18, 24, 25]). For quantifying the redundancy of transportation networks, El-Rashidy and Grant-Muller [25] proposed a redundancy index of various nodes in road networks covering the static aspect of redundancy (i.e., alternative paths) and the dynamic feature of redundancy (i.e., the availability of spare capacity under different network loading and service levels). In their redundancy model, the entropy concept was adopted to measure the configuration of a system and to model the uncertainties inherent in road networks. However, the entropy concept may not be intuitive to travellers, who do care about the existence of optional routes/means. Xu *et al.* [18] developed two network-based measures for systematically characterizing the redundancy of transportation networks: one is travel alternative diversity from travellers' perspective to address the question of "*how many effective redundant alternatives are there for travellers?*" and the other one is network spare capacity from planners' perspective to address the question of "*how much redundant capacity does the network have?*"

The above redundancy studies are mainly conducted for road networks. For public transportation networks, Jenelius and Cats [26] developed a methodology for evaluating the value for robustness and redundancy of extending the public transportation network. The value of redundancy of the network extension was defined as the value of robustness (i.e., the change in passenger welfare under network disruptions in the extended network compared to the baseline network) minus the difference in welfare under normal conditions in the extended network compared to the baseline network. However, *in case of disruptions, the travellers' inconvenience significantly depends on the availability of alternative travel options*, i.e., the amount of redundancy in the network. Yang *et al.* [7] applied the route diversity index (i.e., the simple average number of reasonable routes between all O-D pairs) of Xu *et al.* [18] to analyse the route diversity of the Beijing metro network and to identify the vulnerable stations. Yang *et al.* [7] simply considered the reasonable/efficient routes in defining the route diversity. For improving the realism of route redundancy in metro networks, in this paper, we consider not only efficient routes but also not-too-long routes (termed as effective routes). To a certain degree, this modelling captures both network topology and travellers' route choice behaviours (via the route cost constraint). In addition, the transfer costs at transfer stations are explicitly modelled in metro networks, which is a significant difference between road networks and metro networks.

2.2. Measures of Topology Network Performance. In the past decades, many scholars (e.g., [27–30]) have examined the metro network performances from the viewpoint of graph theory and complex network by focusing on the topological properties of metro networks. The small-world property and scale-free pattern are considered as significant network topological properties, which are also well-studied in metro networks. Small-world effect is referred to as a high clustering and small average shortest path length ([31, 32]), and a scale-free network is defined as the network with a nodal degree distribution following the power law distribution ([32]). For example, Latora and Marchiori [33] analysed the small-world property of Boston subway network; Derrible and Kennedy [34] demonstrated that most metros are small-worlds and scale-free by looking at 33 metro systems.

Besides, many metrics have been proposed to measure the performances of topology networks. According to Grubesic *et al.* [35] and Zhang *et al.* [36], these metrics could be divided to *connectivity and accessibility measures of the network topology*. Typical network performance measures based on topology network are summarized in Table 1. On the one hand, the connectivity measures (e.g., Alpha, Beta, and Gamma indices, average degree, cyclomatic number, and clustering coefficient) are used to assess the connectedness. For these measures, Alpha, Beta, and Gamma indices represent the connectivity and complexity of a network. Specifically, Alpha index is expressed by the ratio of the number of cycles to the maximum number of cycles; Beta index represents the relationship between the number of links and the number of nodes; and Gamma index quantifies the relationship between the number of links and the maximum possible number of links [36]. The node degree measures the number of links converging to each node [37]. The cyclomatic number calculates the number of cycles (or loops), which has been proposed as a topology metric to evaluate the total number of alternatives from the aggregated perspective of the whole network ([34, 36]). The clustering coefficient, known as transitivity, represents an alternative possibility as it measures the overall probability for the network to have interconnected adjacent nodes ([25, 37]). On the other hand, the accessibility measures of network topology, such as diameter and average shortest path length, are directly related to the shortest paths. Network efficiency measures the travel efficiency of passengers between two nodes ([38]). Node betweenness characterizes the centrality of nodes, which could reflect the role of nodes in the network ([39]). These measures have been widely extended to consider passenger flows, route choice, etc. and applied to the robustness and vulnerability analysis of metro networks ([34, 40–46]).

Compared with these typical measures, the number of redundant effective routes in this paper is still based on the topology network. However, it is evaluated for each O-D pair, which can provide disaggregate information (e.g., the number of behaviourally effective routes) explicitly from the travellers' perspective. On the one hand, taking the travel choice behaviour into account, the number of effective routes measures the network redundancy from the disaggregate O-D level, while the connectivity measures listed in Table 1 are based on the number of nodes and edges or aggregated from the node level to the network level. On the other hand, route redundancy represents a set of alternative choices provided to travellers. As travellers might not choose the shortest paths in reality ([30, 47, 48]), the number of effective routes would provide more information than the above accessibility measures based on the shortest paths.

3. Methodology

In this section, we provide the definition, measure, and computational method of route redundancy in metro networks.

3.1. Definition and Measure of Route Redundancy. As summarized in Section 2.1, although there is no consensus definition

Table 1: Typical network performance measures based on topology network ([35–37]).

Metrics		Specifications	Notes
Connectivity	Alpha index	$\alpha = \frac{e - v + 1}{2v - 5}$	e: the number of undirected edges;
	Beta index	$\beta = \frac{e}{v}$	v: the number of nodes;
	Gamma index	$\gamma = \frac{e}{3v - 6}$	d_i: the number of nodes directly connected to node i;
	Average Degree	$\overline{d} = \frac{\sum_i d_i}{v}$	
	Cyclomatic number	$\mu = e - v + 1$	e_i: the number of undirected edges among the nodes directly connected to node i;
	Clustering coefficient	$C = \frac{1}{v}\sum_i \frac{2e_i}{d_i(d_i - 1)}$	
Accessibility	Diameter	$D = \max(l_{ij})$	l_{ij}: the shortest path length from node i to node j;
	Average shortest path length	$L = \frac{\sum_{i,j,i\neq j} l_{ij}}{v(v-1)/2}$	m_{jk}: the number of shortest paths between node j and node k;
	Network efficiency	$E = \frac{2\sum_{i,j,i\neq j} 1/l_{ij}}{v(v-1)}$	
	Average node betweenness	$\overline{B} = \frac{\sum_i \sum_{j,k,j\neq k} m_{jk}(i)/m_{jk}}{v}$	$m_{jk}(i)$: the number of shortest paths between node j and node k through node i.

on the network redundancy, the availability of alternative paths in a network is widely accepted to be a measure of redundancy (e.g., [10, 18, 24, 25]). Xu *et al.* [18] proposed two network-based measures (i.e., travel alternative diversity and network spare capacity) to model transportation network redundancy. They defined travel alternative diversity as the existence of multiple modes and effective routes available for travellers or the number of effective connections between a specific O-D pair. For quantifying the number of alternatives in the metro network, we customize this concept to the route redundancy of metro networks with station transfer costs, i.e., *the number of behaviourally effective routes available for passengers between any two stations in the metro network*. Travellers may not treat all simple paths as their usable paths. Following Xu *et al.* [18], we consider two requirements of effective route: efficient path and not-too-long path.

(1) Efficient Path. Dial [49] defined the efficient path as follows: if a path only consists of links that take network users further away from the origin, it is an efficient path. Consider a directed graph $G = (N, A)$, where N and A are the set of nodes and directed links, and R and S are the set of origins and destinations, respectively. All links on an efficient path should satisfy

$$c_r(a_h) > c_r(a_t), \quad \forall a \in A \tag{1}$$

where $c_r(a_h)$ and $c_r(a_t)$ are the shortest cost from origin r to a_h (the head node of link a) and a_t (the tail node of link a), respectively.

(2) Not-Too-Long Path. Typically, passengers prefer not-too-long paths with an acceptable travel cost as their reasonable alternative paths when the primary or secondary path is not available under disruptions. Hence, the following length constraint [50] is introduced to guarantee that all links are reasonable enough relative to the shortest path.

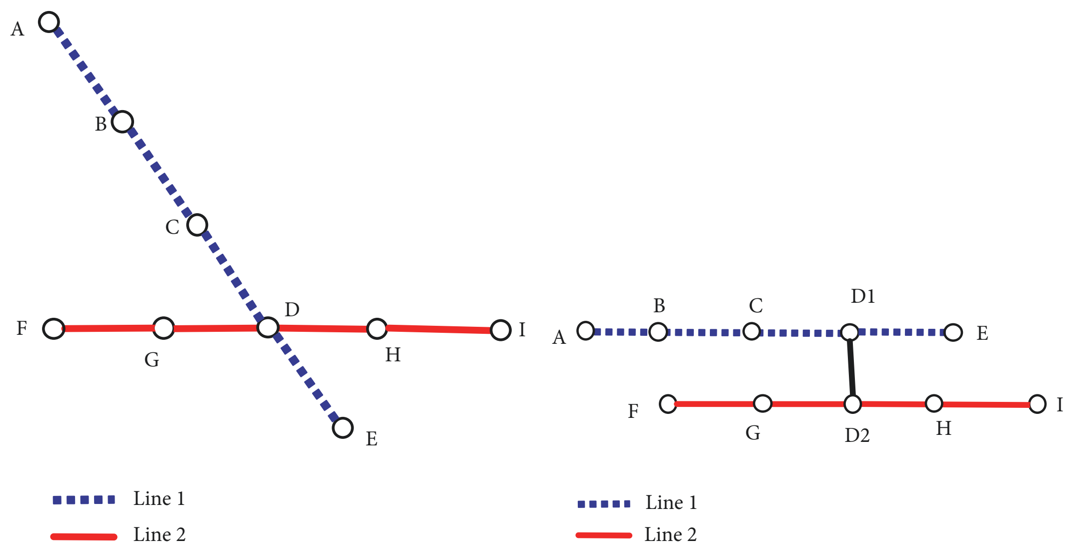
$$c_a \leq (1 + \tau_r^a)(c_r(a_h) - c_r(a_t)), \quad \forall a \in A_k \tag{2}$$

where c_a is the cost (or travel time) of link a; A_k is the set of links on path k; τ_r^a is an allowable or acceptable elongation ratio of considering link a with respect to origin r. By constraining the length of each link, we can guarantee that the total cost of a path has the following upper bound:

$$\begin{aligned} c_k &= \sum_{a\in A_k} c_a \leq \sum_{a\in A_k} (1 + \tau_r^a)(c_r(a_h) - c_r(a_t)) \\ &\leq \sum_{a\in A_k} (1 + \tau_r^{max})(c_r(a_h) - c_r(a_t)) \\ &= (1 + \tau_r^{max})(c_r(s) - c_r(r)) = (1 + \tau_r^{max}) c_r(s) \end{aligned} \tag{3}$$

where c_k is the cost of path k; $c_r(r)$ and $c_r(s)$ are the shortest cost from origin r to r and to destination s, respectively; and τ_r^{max} is the maximum of τ_r^a. According to Leurent [50], τ_r^a can be set as 1.6 for interurban studies or between 1.3 and 1.5 for urban studies. In this paper, we set τ_r^a as 1.5 for each link to constrain the length of paths in metropolitan metro networks. Thus, in the measure of route redundancy, we only consider the paths whose lengths do not exceed 2.5 times the shortest path cost between each O-D pair. Without this requirement, the route redundancy will be significantly overestimated, leading to an optimistic assessment.

When considering the path costs, the transfer time at transfer stations from one line to another line cannot be ignored. Passengers usually prefer paths with fewer interchanges and shorter travel time, so the transfer time would affect passengers' path acceptance or choice. In this study, Space L representation is applied to describe the network



(a) Original network

(b) Representation with virtual stations and links

Figure 1: Metro network representation.

topology of metro networks, which treats stations as nodes and links between consecutive stations as edges [51]. Furthermore, we add virtual stations and links to represent the transfer time. For example, consider the metro network shown in Figure 1(a), which consists of 2 lines, 9 stations, and 8 two-way links. Passengers can transfer between line 1 and line 2 at station D. We can introduce virtual stations D1 and D2, as shown in Figure 1(b), where the link length between D1 and D2 represents the transfer time or disutility at station D. Therefore, the original network G (9, 16) can be represented by a directed graph G (10, 18).

We should point out that passengers still treat the transfer station as a single station. Therefore, we need to map the route redundancy calculated in the represented network back to the route redundancy of the original network by merging the virtual stations. The merging principle is that the number of effective paths between a nontransfer station and a transfer station is equal to the number of effective paths between the nontransfer station and the *nearest* virtual station of the transfer station. For example, the route redundancy of O-D pair (A, D) in Figure 1(a) is represented by the number of effective paths between A and D1 in Figure 1(b), and the route redundancy of O-D pair (F, D) is represented by the number of effective paths between F and D2.

3.2. Computational Method of Route Redundancy. In this section, the computational method of calculating the number of effective paths between each O-D pair is presented. Meng *et al.* [52] developed a polynomial-time combinational algorithm to compute the number of efficient paths between any two nodes without path enumeration and storage. Xu *et al.* [18] further extended it to consider the not-too-long path constraint. This algorithm consists of two parts: (1) constructing the subnetwork $G_r = (N_r, A_r)$ for each origin r, where N_r and A_r are the set of nodes and links in the subnetwork; (2) computing the number of effective paths from origin r to all nodes in the subnetwork. The detailed procedure is presented as follows:

(1) Construct the Subnetwork G_r for Each Origin r. Perform the Dijkstra's shortest path algorithm to obtain the shortest cost $c_r(n)$ from origin r to all the other nodes n ($n \neq r$). Then, remove the links that satisfy $c_r(a_h) \leq c_r(a_t)$ or $c_a > (1 + \tau_r^a)(c_r(a_h)\text{-}c_r(a_t))$ from the link set A. This step is to guarantee the satisfaction of efficient paths and not-too-long paths in defining the route redundancy.

(2) Calculate the Number of Effective Paths from Origin r to All Nodes in the Subnetwork. Firstly, we initialize the node adjacent matrix $U_r(m, n)$ ($m, n \in N_r$) by setting $U_r(m, n) = 0$ for each origin $r \in R$. For the links in the set A_r (i.e., $a \in A_r$), we set $U_r(a_t, a_h) = 1$. Then, we perform the following operation to calculate the number of effective paths between any node pair in N_r.

for $j \in N_r$

  for $m \in N_r \setminus j$

    for $n \in N_r \setminus j \setminus m$

      $U_r(m, n) = U_r(m, n) + U_r(m, j) \times U_r(j, n)$

    end

  end

end

Finally, the number of effective paths between O-D pair (r, s) n^{rs} can be calculated as follows:

for all O-D pair (r, s)

  $n^{rs} = U_r(r, s)$

end

Table 2: Basic information of the four metro networks.

City	Metro system information			# of different types of stations				
	# of lines	# of stations	Length (km)	Non-transfer	Two lines	Three lines	Four lines	Five lines
Shanghai	14	304	*602*	250 (82.24%)	46	7	1	0
Beijing	17	291	546	239 (82.13%)	50	2	0	0
London	13	*381*	569	309 (81.10%)	58	7	6	1
Tokyo	16	257	361	188 (73.15%)	*46*	*14*	*6*	*3*

Note: the station shared by two or more lines is treated as a single station in the third column; the number in the parenthesis represents the proportion of non-transfer stations.

Remark. The above procedure of calculating the number of effective paths in the subnetwork G_r has a polynomial-time complexity, which involves three nested layers. The principle of this algorithm can be explained as follows:

$$n^{rs} = U^1(r,s) + U^2(r,s) + \cdots + U^p(r,s) \tag{4}$$

where U^p is the p-power matrix of the adjacent matrix U, and $U^p(r,s)$ is equal to the number of paths between O-D pair (r, s) consisting of p-1 intermediate nodes on these paths. Therefore, p-1 equals the total number of nodes in the network minus 2. The number of paths between O-D pair (r, s) is equal to the total number of directed paths without intermediate node (i.e., U^1), paths with one intermediate node (i.e., U^2), and paths with two intermediate nodes (i.e., U^3), till paths with p-1 intermediate nodes (i.e., U^p). This guarantees the correctness of the above algorithm for counting all the effective paths between each O-D pair.

Note that the number of effective paths between each O-D pair is the O-D level measure, while the network-level route redundancy should aggregate the distribution of the O-D-level number of effective paths. In this paper, the statistical indicators of the above distribution will be considered to evaluate the network-level route redundancy, e.g., maximum value, minimum value, the average number of effective paths, median and standard variance of the distribution, and the proportion of O-D pairs with only 1 effective path. In addition, instead of the shortest path length, the length of effective alternative paths could also be considered as the accessibility measures.

4. Case Study

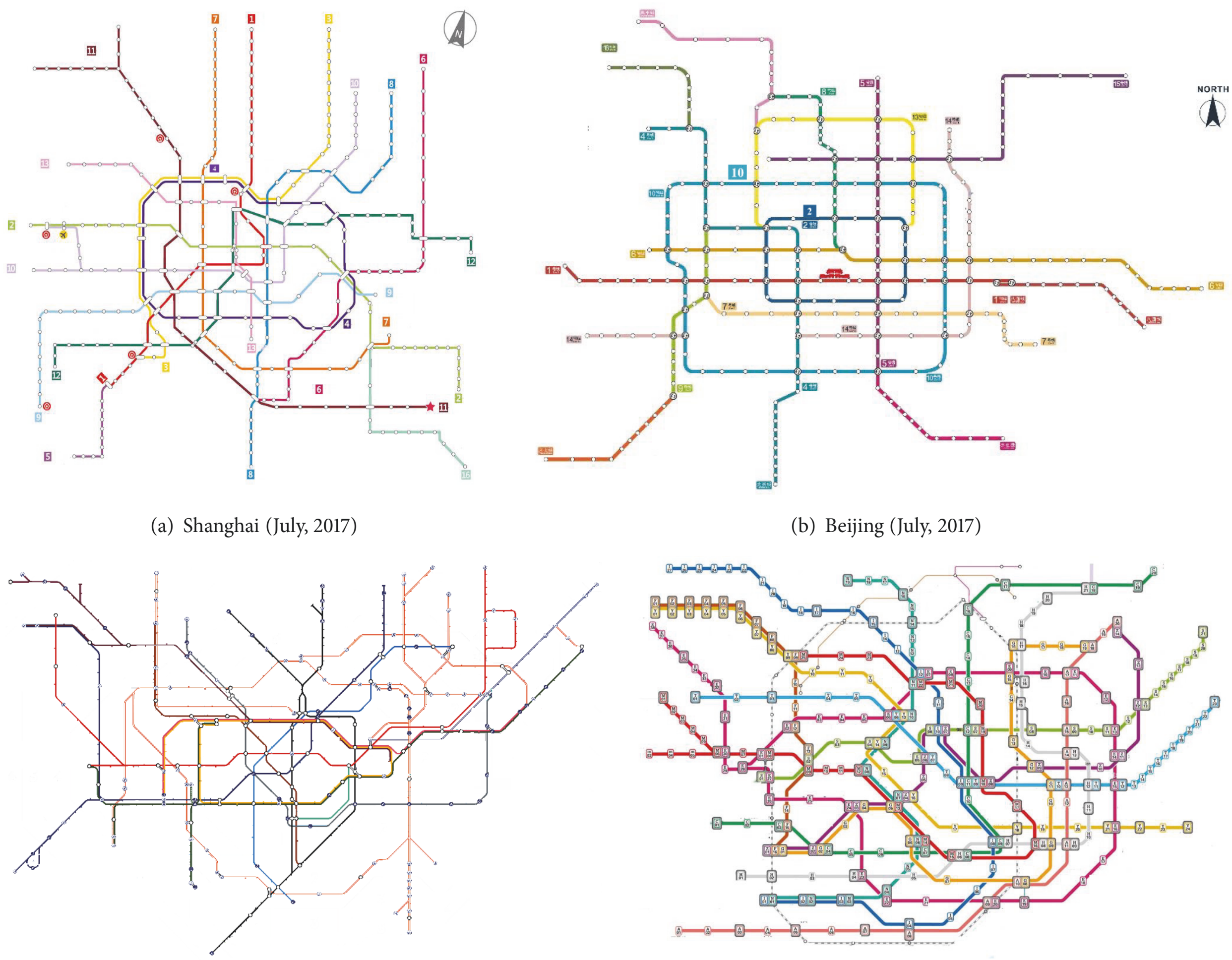
In this section, we examine and compare the route redundancy of four metro networks, i.e., Shanghai, Beijing, London, and Tokyo. Furthermore, the results of topology network measures will be presented, and the differences and relationships between the route redundancy and the typical topology network measures (i.e., the connectivity measures and accessibility measures summarized in Table 1) will be discussed.

4.1. Four Metropolitan Metro Networks. Four well-networked metro systems are analysed in this paper: Shanghai (excluding Maglev line), Beijing (excluding airport line), London (including underground, overground, and Transportation for London (TfL) rail), and Tokyo (including subway and some main private rail lines). These four metro networks are shown in Figure 2. Table 2 further presents the basic information of each metro network. Note that the operation time between any two stations is obtained from their official websites. The transfer time of Shanghai and Beijing metro networks are also obtained from their official websites, and the transfer time of London and Tokyo metro networks is set to 5 min due to the data unavailability, which is the average transfer time of the Shanghai metro network.

According to Table 2, among the four metropolitan cities, the London metro network has the largest number of stations (i.e., 381 stations), while the Shanghai metro network has the largest total line length. The Tokyo metro network has the smallest number of stations as well as the total line length. In terms of transfer stations, the Tokyo network has the most transfer stations among the four networks. The number of nontransfer stations accounts for 73.15% in Tokyo, which is the smallest among the four networks. Meanwhile, there are 6 four-line and 3 five-line transfer stations in the Tokyo network, which are larger than those in the London network, while the Shanghai and Beijing networks do not have five-line transfer stations.

4.2. Route Redundancy Results. Using the computational method in Section 3.2, we can obtain the number of effective paths between each O-D pair. Since the four metro systems have different network scales, the proportion of O-D pairs with each number of effective paths is calculated for a fair comparison. Figure 3 shows the distributions of the number of effective paths among all O-D pairs in each network. Their statistical indicators are also presented in Table 3.

One can see that all O-D pairs in the four metro networks have at least one effective path. The Tokyo metro network has at most 58 effective paths between two stations, while the Beijing metro network only has at most 14 effective paths between two stations and Shanghai has at most 13 effective paths. The Beijing and Shanghai metro networks have more than 70% of the O-D pairs with only 1 effective path, while Tokyo has 66.23% and London has 56.60% of the O-D pairs with only 1 effective path. This means that passengers of more than 70% of the O-D pairs have no other effective alternatives for their metro trips in Shanghai and Beijing, while more than 40% of the O-D pairs in the London network and more than 30% of the O-D pairs in the Tokyo network have other effective alternatives. Also, the average numbers of effective paths in the Shanghai and Beijing networks are much smaller than 2 (i.e., 1.47 and



(a) Shanghai (July, 2017)

(b) Beijing (July, 2017)

(c) London (July, 2017)

(d) Tokyo (July, 2017)

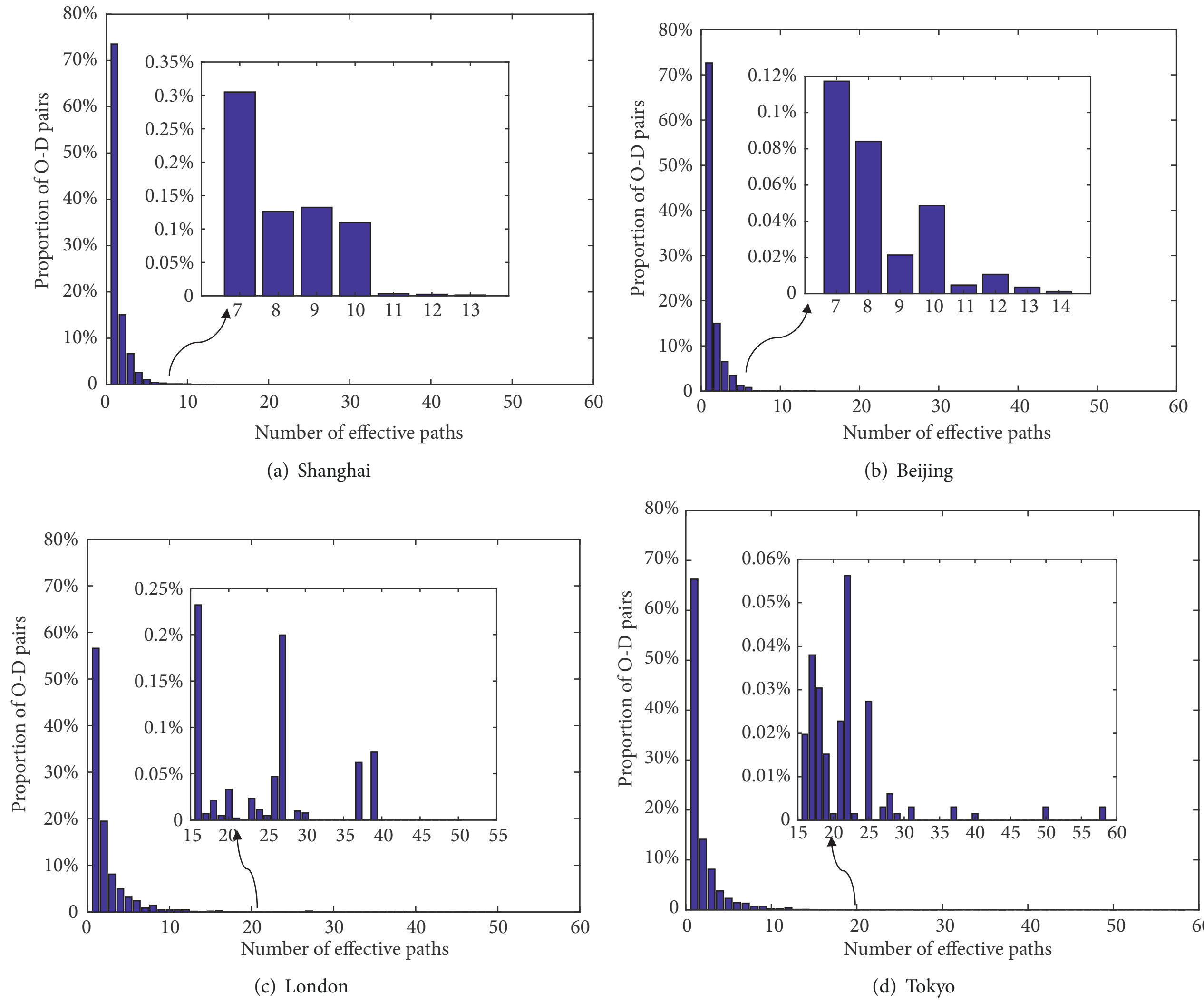
Figure 2: Topologies of the four metropolitan metro networks.

Table 3: Statistical indicators of route redundancy distribution.

City	Max	Min	Median	Average	Standard deviation	Proportion of O-D pairs with only 1 effective path
Shanghai	13	1	1	1.47	1.0258	73.55%
Beijing	14	1	1	1.50	1.0207	72.64%
London	50	1	1	2.31	2.8962	56.60%
Tokyo	58	1	1	1.93	2.0779	66.23%

1.50), while there are 2.31 and 1.93 in London and Tokyo, respectively. The Shanghai and Beijing metro networks have a more concentrated distribution of the number of effective paths as indicated by their smaller average value and standard deviation, while the London and Tokyo metro networks have more diversely distributed route redundancy with a larger average value and standard deviation. As we mentioned before, the existence of more effective paths between an O-D pair means that there are more rerouting opportunities under both normal operations and disruptive events. The travellers' inconvenience generated by disruptions and congestions can be reduced by the availability of alternative effective paths, i.e., route redundancy in the network. Overall, the London metro network has more route redundancy (i.e., predisaster preparedness) than the Tokyo metro network; the Shanghai and Beijing metro networks have similar performance of route redundancy, which have less route redundancy than the London and Tokyo metro networks.

The number of effective paths between any two stations characterizes the route redundancy of metro networks at the O-D level. It is also meaningful to know how many effective paths originating from a station and going to a station. Therefore, the effective connection of each station can be measured. This information is related to the accessibility of metro stations and can be used in business and residential location decisions. We further define the *station-level route*



(a) Shanghai

(b) Beijing

(c) London

(d) Tokyo

Figure 3: Distributions of the number of effective paths (the histogram in the middle of each figure shows the detailed distribution of the number of effective paths).

redundancy index (SRI) as the average number of effective paths originating from and going to a station. As there is at least one effective path from a station and one effective path to a station, SRI is not less than 2. The formulation of SRI is as follows:

$$SRI_i = \frac{\left(\sum_{j=1,j\neq i}^{|N|} n_{ij} + \sum_{j=1,j\neq i}^{|N|} n_{ji}\right)}{(|N|-1)} \quad (5)$$

where n_{ij} is the number of effective paths from station i to station j; $|N|$ is the total number of stations in the metro network. The distributions of SRI of the four networks are shown in Figure 4. We can observe that the station-level route redundancy of the Shanghai and Beijing metro networks are highly concentrated in the interval of $[2, 4]$ with an average SRI of 2.95 and 2.99, respectively. The London and Tokyo metro networks have a larger range of SRI (i.e., from 2 to 11 and from 2 to 8). Both of London and Tokyo are mostly concentrated in the interval of $[3, 4]$, but the London network has more stations with SRI being greater than 5. Therefore, the average SRI of London (i.e., 4.62) is the highest among the four cities. Overall, the distribution of SRI is consistent with the distribution of the O-D-level number of effective paths shown in Figure 3 and Table 3. This further verifies the promising route redundancy performance of the London and Tokyo metro networks.

4.3. Comparison of Route Redundancy and Other Measures. The results in Section 4.2 show that the London metro network has the best performance in the O-D-level and station-level route redundancy compared with the Tokyo, Shanghai, and Beijing metro networks. As summarized in Table 1, the typical network performance measures based on the topology network can be divided into two categories: connectivity measures and accessibility measures. The route redundancy is also based on the topology metro network. Below we discuss the difference and relationship between the route redundancy and these typical measures using the above four metro networks.

Table 4: Connectivity measures of the four metro networks.

City	Alpha index	Beta index	Gamma index	Average Degree	Cyclomatic number	Clustering coefficient
Shanghai	0.0796	1.1546	0.3874	2.3092	48	0.0081
Beijing	0.0711	1.1375	0.3818	2.2749	41	0.0023
London	0.0753	1.1470	0.3843	2.2940	57	0.0103
Tokyo	0.1297	1.2529	0.4209	2.5058	66	0.0214

Table 5: Accessibility measures of the four metro networks.

City	Diameter (min)	Average shortest path length (min)	Network efficiency (min^{-1})	Average node betweenness
Shanghai	152	23.7878	0.0159	4,719
Beijing	142	23.4021	0.0164	4,496
London	127	23.0497	0.0155	6,641
Tokyo	77	14.9963	0.0955	4,138

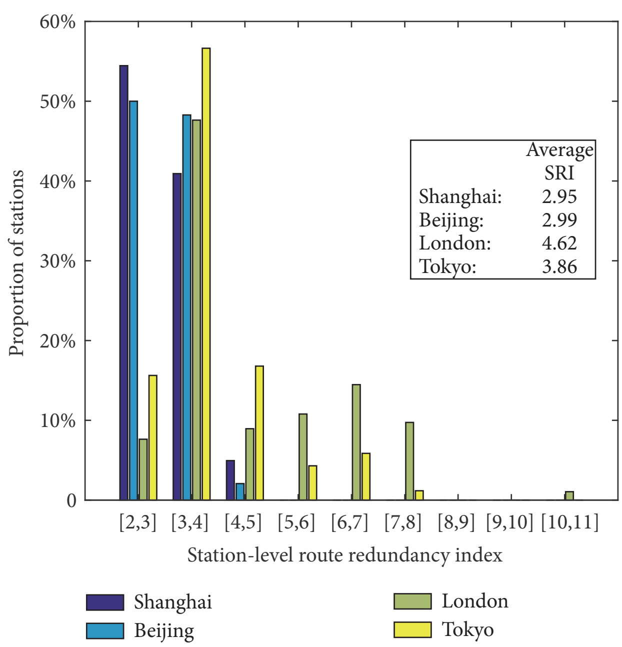


Figure 4: Distributions of station-level route redundancy index.

(1) Route Redundancy versus Connectivity Measures. Table 4 presents the results of connectivity measures of the four metro networks. In terms of the Alpha index, Beta index, and Gamma index, the decreasing sequence is Tokyo, Shanghai, London, and Beijing, which means that the Tokyo metro network has the best connectivity among the four networks. Among the three indices, Alpha index is equal to the ratio of the number of cycles to the maximum possible number of cycles in a network. The result of the cyclomatic number shows that the Tokyo network has the largest number of cycles (i.e., 66 cycles) among the four metro networks, and London has more cycles than Shanghai and Beijing. The result difference between the Alpha index and the cyclomatic number is due to the larger number of stations in the London metro network than Shanghai and Beijing. The average degree and clustering coefficient are both node-level metrics. For a single line, the degree of most of stations is 2 except for the beginning and ending nodes. The degree of transfer stations in a network is usually greater than 2. The average degrees of the four metro networks are all between 2 and 3, and the largest one is also in the Tokyo network as the transfer stations account for the largest proportion. The clustering coefficient describes the degree to which nodes in the network tend to cluster together. The results show that the clustering coefficient of the four networks are all close to 0. The Tokyo network is the highest, and the nodes of the London network are more clustered than Shanghai and Beijing. Overall, the Tokyo metro network has the best connectivity, which is the most complex one among the four metro networks. The values of connectivity measures of the Beijing metro network are the lowest. The London metro network has more cycles and its nodes are more clustered than Shanghai, but the other four measures indicate that Shanghai has slightly better connectivity than London.

Among the typical measures of network connectivity, Alpha index, Beta index, Gamma index, and the cyclomatic number are calculated based on the number of nodes and the number of edges; the average degree and the clustering coefficient are aggregated from the node-level metrics to the network level. These measures are mainly related to the network structure. In contrast, the route redundancy measure proposed in this paper is able to consider passengers' route choice besides the network topology and is aggregated from the O-D level, which could explicitly reflect alternative routes information (besides the shortest path only) for each O-D pair. Therefore, although the London metro network does not have the best topological connectivity as shown in Table 4, it could provide more alternative routes for travellers than the other three metro networks as shown in Section 4.2.

(2) Route Redundancy versus Accessibility Measures. The results of accessibility measures are provided in Table 5. As introduced in Section 2.2, diameter and average shortest path length are directly related to the shortest paths. The Shanghai metro network has the largest diameter (i.e., 152), which is about two times the Tokyo metro network. Both Beijing and London also have diameters greater than 100. The average

shortest path length of Tokyo is the lowest (i.e., 14.9963) and the other three networks have the average shortest path length, greater than 23. This means that travellers of each O-D pair should take at least an average of 23 minutes in Shanghai, Beijing, and London metro networks. Therefore, the network efficiency of the Tokyo network is much higher than the other three networks, as network efficiency is calculated by the harmonic mean of the length of all the shortest paths in the network. Compared to the above three measures, the average node betweenness is a node-level measure quantifying the importance of each node, which is also related to the shortest paths. For the London metro network, each node would be passed by the shortest paths of around 6,641 O-D pairs on average, while this number is less than 5,000 for the other three networks. It indicates that the nodes in the London metro network tend to be more important as they would be passed by more shortest paths. In summary, similar to the connectivity results, the Tokyo metro network also has the best accessibility performance among the four networks, as shown by the smaller diameter, shorter path length, and higher network efficiency. As to the average node betweenness, the nodes in the Tokyo metro network are also less important than the other three networks. The Shanghai, Beijing, and London networks have a similar accessibility performance but the nodes of the London metro network tend to be more important.

The accessibility measures, i.e., diameter, average shortest path length, network efficiency, and node betweenness, are calculated based on the shortest path of each O-D pair (Table 5). Empirical studies (e.g., [30, 47, 48]) have shown that travellers may not always select the shortest path due to many reasons such as perception error and multicriterion consideration (e.g., [53]). Considering this behaviour realism, this paper explicitly captures the number of effective alternative paths with travel costs constraints. Although the number of effective paths cannot directly reflect the distance or efficiency of a network, the concept of effective path could be further applied to measure the network accessibility by replacing the shortest paths. Therefore, although the Tokyo metro network has the best accessibility, it has fewer effective alternatives for travellers than the London metro network.

Overall, the results of connectivity and accessibility measures are quite different from the results of route redundancy. Among the four metro networks, the Tokyo metro network has better connectivity and accessibility, while the London metro network has more route redundancy. The main reason is that the route redundancy measure considers the O-D-level travellers' route choice (i.e., efficient route and not-too-long route) beyond the pure network topology (as in the connectivity measures) and the shortest path (as in the accessibility measures) considerations.

5. Conclusions

In this paper, we provided a new dimension of metro network performance explicitly from travellers' perspective—route redundancy/diversity. The route redundancy was defined as the number of behaviourally effective paths (i.e., efficient and not-too-long paths) between two stations, which reflects the predisaster preparedness of the metro network. Based on the O-D level route redundancy, we further defined the station-level route redundancy as the average number of effective paths originating from or going to a station. For presenting the characteristics of this measure, route redundancy of four dense metropolitan metro networks, i.e., Shanghai, Beijing, London, and Tokyo, were examined. As the route redundancy proposed in this paper is still based on the topology network, the results of typical measures of topology network performance, including connectivity and accessibility, were presented to discuss the differences and relationships between the route redundancy and these typical measures.

The route redundancy results indicated that the London metro network has the best route redundancy performance in terms of the mean and standard deviation of the number of effective paths between an O-D pair as well as the proportion of O-D pairs with only 1 effective path. The average number of effective paths of the Tokyo network is less than London, even if the maximum number of effective paths of Tokyo is the largest. In case of normal operations or disruptions, the London metro network would provide the most rerouting opportunities for travellers among the four networks. Both of the Shanghai and Beijing metro networks have less route redundancy than the London and Tokyo metro networks, as there are fewer effective alternatives for travellers in the Shanghai and Beijing networks. According to the results of typical topology network performance measures, the Tokyo metro network has the best connectivity and accessibility performances among the four networks. The differences between the route redundancy and these typical connectivity and accessibility measures are attributed to the following two reasons: on the one hand, compared with the measures based on the number of nodes and edges or aggregated from the node level to the network level, route redundancy provides disaggregate O-D level information and takes the travellers' route choice behaviour into account. On the other hand, compared with the measures related to the shortest paths, the route redundancy explicitly captures the number of behaviourally effective alternative paths with travel costs constraints, which is more consistent with the empirical observation that in reality travellers may not always select the shortest path.

There are some limitations in this paper that should be addressed in future research.

(i) Based on the data of actual disruptions or agent-based simulations, the route redundancy measure proposed in this paper could be applied to examine the network performance under disruptions and to justify the vulnerability analysis results of metro networks. Using the concept of route redundancy, we can further identify which stations (i.e., critical stations) have the largest impact on the route redundancy of a metro network (e.g., [54, 55]). The identification of critical stations could assist in the resource allocation for strategically protecting the metro network and the decision-making for emergency evacuation planning.

(ii) It is interesting to examine the redundancy of a multi-modal transportation network, which would provide not only alternative routes (i.e., route redundancy proposed in this paper) but also alternative travel modes for travellers.

(iii) This paper only examined the route redundancy dimension of network redundancy. To provide a more comprehensive assessment, the network capacity dimension of metro network redundancy could also be modelled and examined by considering both travellers' and planners' perspectives.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request. The networks and operation time of Beijing, Shanghai, London, and Tokyo metro systems are accessed at http://www.bjsubway.com, http://service.shmetro.com, https://tfl.gov.uk/, and https://www.tokyometro.jp/index.html, respectively.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (71890970/71890973) and Fundamental Research Funds for the Central Universities of China. These supports are gratefully acknowledged.

References

[1] A. González-Gil, R. Palacin, P. Batty, and J. P. Powell, "A systems approach to reduce urban rail energy consumption," *Energy Conversion and Management*, vol. 80, pp. 509–524, 2014.

[2] "Wikipedia, "Beijing Subway,"," https://en.wikipedia.org/wiki/Beijing_Subway, 2018.

[3] "Wikipedia, "List of metro systems,"," https://en.wikipedia.org/wiki/List_of_metro_systems, 2018.

[4] "Wikipedia, "Shanghai metro,"," https://en.wikipedia.org/wiki/Shanghai_Metro, 2018.

[5] X. Ding, S. Guan, D. J. Sun, and L. Jia, "Short turning pattern for relieving metro congestion during peak hours: the substance coherence of Shanghai, China," *European Transport Research Review*, vol. 10, no. 2, p. 28, 2018.

[6] G. Nian, F. Chen, Z. Li et al., "Evaluating the alignment of new metro line considering network vulnerability with passenger ridership," *Transportmetrica A: Transport Science*, 2019.

[7] X. Yang, A. Chen, and B. Ning, "Measuring route diversity for urban rail transit networks: a case study of the beijing metro network," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 2, pp. 259–268, 2017.

[8] S. C. Calvert and M. Snelder, "A methodology for road traffic resilience analysis and review of related concepts," *Transportmetrica A: Transport Science*, vol. 14, no. 1-2, pp. 130–154, 2017.

[9] L.-G. Mattsson and E. Jenelius, "Vulnerability and resilience of transport systems - A discussion of recent research," *Transportation Research Part A: Policy and Practice*, vol. 81, pp. 16–34, 2015.

[10] M. A. P. Taylor, *Vulnerability Analysis for Transportation Networks*, Elsevier, 2017.

[11] A. Reggiani, P. Nijkamp, and D. Lanzi, "Transport resilience and vulnerability: the role of connectivity," *Transportation Research Part A: Policy and Practice*, vol. 81, no. 11, pp. 4–15, 2015.

[12] The White House, *Critical Infrastructure Security and Resilience*, Office of the Press Secretary, The White House, Washington, DC, USA, 2013, http://www.fas.org/irp/offdocs/ppd/ppd-21.pdf.

[13] K. E. Worton, "Using socio-technical and resilience frameworks to anticipate threat," in *Proceedings of the Workshop on Socio-Technical Aspects on Security and Thrust*, pp. 19–26, 2012.

[14] M. Bruneau, S. E. Chang, R. T. Eguchi et al., "A framework to quantitatively assess and enhance the seismic resilience of communities," *Earthquake Spectra*, vol. 19, no. 4, pp. 733–752, 2003.

[15] A. Reggiani, "Network resilience for transport security: Some methodological considerations," *Transport Policy*, vol. 28, pp. 63–68, 2013.

[16] M. Ouyang, L. Dueñas-Osorio, and X. Min, "A three-stage resilience analysis framework for urban infrastructure systems," *Structural Safety*, vol. 36-37, pp. 23–31, 2012.

[17] S. Lhomme, D. Serre, Y. Diab, and R. Laganier, "Analyzing resilience of urban networks: A preliminary step towards more flood resilient cities," *Natural Hazards and Earth System Sciences*, vol. 13, no. 2, pp. 221–230, 2013.

[18] X. Xu, A. Chen, S. Jansuwan, C. Yang, and S. Ryu, "Transportation network redundancy: Complementary measures and computational methods," *Transportation Research Part B: Methodological*, vol. 114, pp. 68–85, 2018.

[19] V. P. Singh and J. Oh, "A Tsallis entropy-based redundancy measure for water distribution networks," *Physica A: Statistical Mechanics and its Applications*, vol. 421, pp. 360–376, 2015.

[20] L. Schäfer, S. García, A. Mitschke, and V. Srithammavanh, "Redundancy system design for an aircraft door management system," *Computers & Operations Research*, vol. 94, pp. 11–22, 2018.

[21] K. Berdica, "An introduction to road vulnerability: what has been done, is done and should be done," *Transport Policy*, vol. 9, no. 2, pp. 117–127, 2002.

[22] B. Immers, J. Stada, and I. Yperman, "Towards robust road network structures," *Slovak Journal of Civil Engineering*, vol. 4, pp. 10–17, 2004.

[23] D. Freckleton, K. Heaslip, W. Louisell et al., "Evaluation of transportation network resiliency with consideration for disaster magnitude," in *Proceedings of the Paper presented at the 91st annual meeting of the Transportation Research Board*, vol. 10, Washington, DC, USA, 2012.

[24] M. Snelder, H. J. van Zuylen, and L. H. Immers, "A framework for robustness analysis of road networks for short term variations in supply," *Transportation Research Part A: Policy and Practice*, vol. 46, no. 5, pp. 828–842, 2012.

[25] R. A. El-Rashidy and S. Grant-Muller, "The evaluation of redundancy for road traffic networks," *Transport*, vol. 31, no. 4, pp. 427–439, 2016.

[26] E. Jenelius and O. Cats, "The value of new public transport links for network robustness and redundancy," *Transportmetrica A: Transport Science*, vol. 11, no. 9, pp. 819–835, 2015.

[27] D. Gattuso and E. Miriello, "Compared analysis of metro networks supported by graph theory," *Networks and Spatial Economics*, vol. 5, no. 4, pp. 395–414, 2005.

[28] S. Derrible and C. Kennedy, "Characterizing metro networks: State, form, and structure," *Transportation*, vol. 37, no. 2, pp. 275–297, 2010.

[29] C. Roth, S. M. Kang, M. Batty, and M. Barthelemy, "A long-time limit for world subway networks," *Journal of the Royal Society Interface*, vol. 9, no. 75, pp. 2540–2550, 2012.

[30] X. Wu, H. Dong, C. K. Tse, I. W. Ho, and F. C. Lau, "Analysis of metro network performance from a complex network perspective," *Physica A: Statistical Mechanics and its Applications*, vol. 492, pp. 553–563, 2018.

[31] D. J. Watts, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[32] A. Barabasi and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[33] V. Latora and M. Marchiori, "Is the Boston subway a small-world network?" *Physica A: Statistical Mechanics and its Applications*, vol. 314, no. 1, pp. 109–113, 2002.

[34] S. Derrible and C. Kennedy, "The complexity and robustness of metro networks," *Physica A: Statistical Mechanics and its Applications*, vol. 389, no. 17, pp. 3678–3691, 2010.

[35] T. H. Grubesic, T. C. Matisziw, A. T. Murray, and D. Snediker, "Comparative approaches for assessing network vulnerability," *International Regional Science Review*, vol. 31, no. 1, pp. 88–112, 2008.

[36] X. Zhang, E. Miller-Hooks, and K. Denny, "Assessing the role of network topology in transportation network resilience," *Journal of Transport Geography*, vol. 46, pp. 35–45, 2015.

[37] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D. W. Hwang, "Complex networks: Structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, 2006.

[38] M. Zanin, X. Sun, and S. Wandelt, "Studying the topology of transportation systems through complex networks: handle with care," *Journal of Advanced Transportation*, vol. 2018, pp. 1–17, 2018.

[39] W. Liu, X. Li, T. Liu, and B. Liu, "Approximating betweenness centrality to identify key nodes in a weighted urban complex transportation network," *Journal of Advanced Transportation*, vol. 2019, Article ID 9024745, pp. 1–8, 2019.

[40] J. Zhang, X. Xu, L. Hong, S. Wang, and Q. Fei, "Networked analysis of the Shanghai subway network, in China," *Physica A: Statistical Mechanics and its Applications*, vol. 390, no. 23-24, pp. 4562–4570, 2011.

[41] O. Cats and E. Jenelius, "Dynamic vulnerability analysis of public transport networks: mitigation effects of real-time information," *Networks and Spatial Economics*, vol. 14, no. 3-4, pp. 435–463, 2014.

[42] Y. Yang, Y. Liu, M. Zhou, F. Li, and C. Sun, "Robustness assessment of urban rail transit based on complex network theory: A case study of the Beijing Subway," *Safety Science*, vol. 79, pp. 149–162, 2015.

[43] X. Wang, Y. Koc, and S. Derrible, "Quantifying the robustness of metro networks," in *Proceedings of the Presented at the 6th International Symposium on Transportation Network Reliability*, Japan, 2015.

[44] D. J Sun, Y. Zhao, and Q. Lu, "Vulnerability analysis of urban rail transit networks: A case study of Shanghai, China," *Sustainability* , vol. 7, no. 6, pp. 6919–6936, 2015.

[45] D. J. Sun and S. Guan, "Measuring vulnerability of urban metro network from line operation perspective," *Transportation Research Part A: Policy and Practice*, vol. 94, pp. 348–359, 2016.

[46] L. Sun, Y. Huang, Y. Chen, and L. Yao, "Vulnerability assessment of urban rail transit based on multi-static weighted method in Beijing, China," *Transportation Research Part A: Policy and Practice*, vol. 108, pp. 12–24, 2018.

[47] V. V. Dixit and L. Denant-Boemont, "Is equilibrium in transport pure Nash, mixed or Stochastic?" *Transportation Research Part C: Emerging Technologies*, vol. 48, pp. 301–310, 2014.

[48] W. Tang and D. M. Levinson, "Deviation between actual and shortest travel time paths for commuters," *Journal of Transportation Engineering Part A: Systems*, vol. 144, no. 8, 2018.

[49] R. B. Dial, "A probabilistic multipath traffic assignment model which obviates path enumeration," *Transportation Research*, vol. 5, no. 2, pp. 83–111, 1971.

[50] F. M. Leurent, "Curbing the computational difficulty of the logit equilibrium assignment model," *Transportation Research Part B: Methodological*, vol. 31, no. 4, pp. 315–326, 1997.

[51] C. Von Ferber, T. Holovatch, Y. Holovatch, and V. Palchykov, "Network harness: metropolis public transport," *Physica A: Statistical Mechanics and its Applications*, vol. 380, no. 1-2, pp. 585–591, 2007.

[52] Q. Meng, D. Lee, and R. L. Cheu, "Counting the different efficient paths for transportation networks and its applications," *Journal of Advanced Transportation*, vol. 39, no. 2, pp. 193–220, 2005.

[53] X. Xu, A. Chen, Z. Zhou, and L. Cheng, "A multi-class mean-excess traffic equilibrium model with elastic demand," *Journal of Advanced Transportation*, vol. 48, no. 3, pp. 203–222, 2014.

[54] W. Jing, X. Xu, and Y. Pu, "A route diversity-based vulnerability analysis of metro networks without reevaluating disruption scenarios," Working paper, 2019.

[55] X. Xu, A. Chen, and C. Yang, "An optimization approach for deriving upper and lower bounds of transportation network vulnerability under simultaneous disruptions of multiple links," *Transportation Research Part C: Emerging Technologies*, vol. 94, pp. 338–353, 2018.